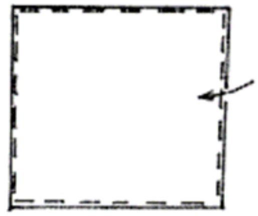
PROBLEM 4.1

KNOWN: Air leaks into an initially evacuated tank at a constant mass flow rate

FIND: Determine the pressure in the tank after 30 s.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume has one inlet and no exits. (2) The air behaves as an ideal gas. (3) The mass flow rate is constant.

ANALYSIS: The mass rate balance, Eq. 4.2, reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i \Rightarrow dm_{cv} = \dot{m}_i \, dt$$

Integrating

$$\int_0^m dm_{cv} = \int_0^{30} \dot{m}_i \, dt = \int_0^{30} (0.004) \, dt$$

$$m(t=30) = (0.004)(30) = 0.12 \text{ lb}$$

Now, using the ideal gas model

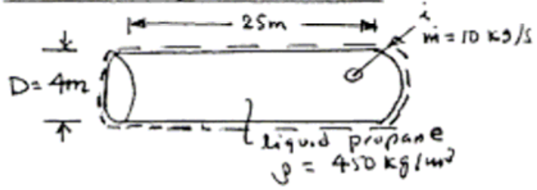
$$P_2 = \frac{mRT_2}{V} = \frac{(0.12 \text{ lb})\left(\frac{1545}{28.97} \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(530°\text{R})}{(5 \text{ ft}^3)} \left|\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right|$$

$$= 4.711 \text{ lbf/in}^2 \longleftarrow P_2$$

PROBLEM 4.2

Liquid propane enters an initially-empty cylindrical storage tank at a mass flow rate of 10 kg/s. The tank is 25-m long and has a 4-m diameter. The density of the liquid propane is 450 kg/m^3. Determine the time, in h, to fill the tank.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume has one inlet and no exits. (2) The mass flow rate is constant. (3) The propane density is constant.

ANALYSIS:

The mass rate balance, Eq. 4.2, reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i \Rightarrow m_{cv}(t_f) - m_{cv}(0) = \int_0^{t_f} \dot{m}_i \, dt$$

Since the tank is initially empty and the mass flow rate is constant, this becomes

$$m_{cv}(t_f) = \dot{m}_i t_f \Rightarrow t_f = \frac{m_{cv}(t_f)}{\dot{m}_i}$$

where

$$m_{cv}(t_f) = \rho V = \left(450 \frac{kg}{m^3}\right)\left(\frac{\pi (4m)^2}{4}(25m)\right) = 141,372 \text{ kg}$$

Finally

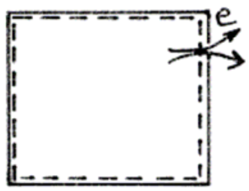
$$t_f = \frac{141,372 \text{ kg}}{(10 \text{ kg/s})}\left|\frac{1 \text{ h}}{3600 \text{ s}}\right| = 3.93 \text{ h}$$

PROBLEM 4.3

KNOWN: A tank containing Ammonia develops a leak. The ammonia escapes at a constant mass flow rate.

FIND: Determine the time at which half the mass has escaped and the pressure in the tank at that time.

SCHEMATIC & GIVEN DATA:



$\dot{m}_e = 0.04$ kg/s
$V = 0.5\ m^3$
$T_1 = 40°C$
$P_1 = 8$ bar
$m_2 = \frac{1}{2} m_1$

ENGR. MODEL: (1) The control volume is shown on the accompanying diagram. (2) The temperature in the tank remains constant. (3) The volume is constant.

ANALYSIS: Applying the mass rate balance

$$\frac{dm_{cv}}{dt} = -\dot{m}_e = -0.04 \text{ kg/s}$$

Integrating over time

$$\int_{m_{cv}(0)}^{m_{cv}(t)} dm_{cv} = -\int_0^t \dot{m}_e\, dt = -0.04\ t$$

Solving for t

$$t = \frac{m_{cv}(t) - m_{cv}(0)}{(-0.04 \text{ kg/s})}$$

From Table A-15, $v_1 = 0.17720\ m^3/kg$. Thus, the initial mass is

$$m_{cv}(0) = \frac{V}{v_1} = 2.822 \text{ kg}$$

and $$m_{cv}(t) = \frac{1}{2} m_{cv}(0) = 1.411 \text{ kg}$$

Thus

$$t = \frac{(1.411 - 2.822) \text{ kg}}{(-0.04 \text{ kg/s})} = 35.3 \text{ s} \longleftarrow t$$

Now, to get P_2, determine v_2

$$v_2 = \frac{V}{m_{cv}(t)} = 0.3544\ m^3/kg$$

Interpolating in Table A-15 at $T_2 = 40°C$, $v_2 = 0.3544\ m^3/kg$

$$P_2 \approx 4.17 \text{ bar} \longleftarrow P_2$$

PROBLEM 4.4

KNOWN: Data are provided for a crude oil storage tank.

FIND: After 24h, determine the mass and volume of oil in the tank.

SCHEMATIC & GIVEN DATA:

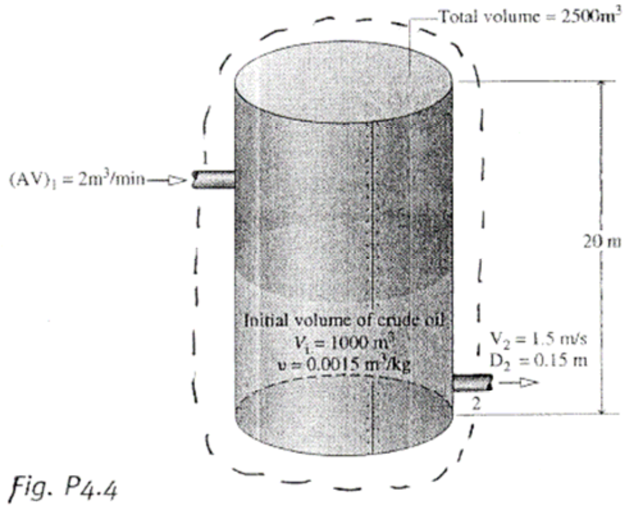


Fig. P4.4

ENGR. MODEL

1. As shown by the sketch, a control volume encloses the storage tank.
2. The specific volume of the oil is constant.

(a) Mass rate balance: $\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2$, where

$$\dot{m}_1 = \frac{(AV)_1}{v} = \left(\frac{2\ m^3/min}{0.0015\ m^3/kg}\right)\left|\frac{60\ min}{1\ h}\right| = 8 \times 10^4\ \frac{kg}{h}$$

$$\dot{m}_2 = \frac{A_2 V_2}{v} = \frac{(\pi D_2^2/4)(V_2)}{v} = \frac{\pi (0.15\,m)^2(1.5\,m/s)}{4\,(0.0015\,m^3/kg)}\left|\frac{3600\,s}{1\,h}\right| = 6.36 \times 10^4\ \frac{kg}{h}$$

$$\therefore \frac{dm_{cv}}{dt} = 1.64 \times 10^4\ \frac{kg}{h}$$

Integrating

$$\Rightarrow \quad m_{cv} - m_{cv}(0) = \left(1.64 \times 10^4\ \frac{kg}{h}\right)(24h) = 39.36 \times 10^4\ kg$$

$$\left(= \frac{V_i}{v} = \left[\frac{1000\ m^3}{0.0015\ m^3/kg}\right] = 66.67 \times 10^4\ kg \right.$$

So,

$$m_{cv}(24h) = (66.67 + 39.36) \times 10^4\ kg = 1.06 \times 10^6\ kg \quad \longleftarrow m_{cv}$$

(b)

$$V(24h) = v\, m_{cv}(24h) = \left(0.0015\ \frac{m^3}{kg}\right)(1.06 \times 10^6\ kg) = 1590\ m^3 \quad \longleftarrow V$$

PROBLEM 4.5

KNOWN: Data are provided for a vegetable oil-filled spray can.

FIND: Determine the mass flow rate per spray, and the mass remaining in the can after a specified number of sprays.

SCHEMATIC & GIVEN DATA:

- 560 sprays, each lasting 0.25 s and having a mass of 0.25 g
- initial mass of vegetable oil in the can is 170 g

ENGR. MODEL: The control volume is shown in the accompanying schematic.

ANALYSIS: (a) Since each spray has a duration of 0.25 s and consists of 0.25 g

$$\dot{m}_e = \frac{0.25\,g}{0.25\,s} = 1\ g/s$$

(b) The mass rate balance, Eq. 4.2, gives on integration

$$m_{cv}(t+\Delta t) - m_{cv}(0) = \cancel{m_i}^{\,0} - m_e$$

where $m_{cv}(0)$ is the initial amount of mass within the can and m_e is the amount of mass that exits. Thus with $m_{cv}(0) = 170\ g$ and

$$m_e = (560\ \text{sprays})\left(\frac{0.25\,g}{\text{spray}}\right) = 140\,g$$

The mass of vegetable oil remaining in the can after 560 sprays is

$$m_{cv} = m_{cv}(0) - m_e$$
$$= 170\,g - 140\,g = 30\,g$$

PROBLEM 4.6

Figure P4.6 shows a mixing tank initially containing 3000 lb of liquid water. The tank is fitted with two inlet pipes, one delivering hot water at a mass flow rate of 0.8 lb/s and the other delivering cold water at a mass flow rate of 1.3 lb/s. Water exits through a single exit pipe at a mass flow rate of 2.6 lb/s. Determine the amount of water, in lb, in the tank after one hour.

SCHEMATIC & GIVEN DATA:

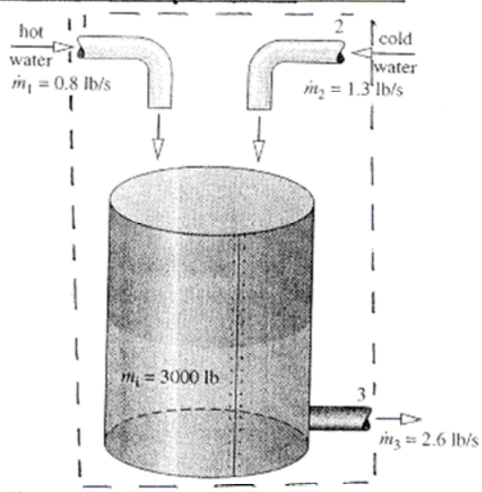


Fig. P4.6

ENGR. MODEL:

1. The control volume has two inlets and one exit, as shown in the sketch.
2. The mass flow rates are constant.

ANALYSIS:

The mass rate balance, Eq. 4.2, reads

$$\frac{dm_{cv}}{dt} = \dot{m}_1 + \dot{m}_2 - \dot{m}_3$$

$$= (0.8 + 1.3 - 2.6)\ \text{lb/s}$$

$$= -0.5\ \text{lb/s}$$

Integrating from $t = 0$ to $t = 1h\ (3600s)$,

$$m_{cv}(1h) - m_{cv}(0) = \left(0.5\ \frac{lb}{s}\right)(3600s)$$

$$= -1800\ \text{lb}$$

$$\therefore\ m_{cv}(1h) = m_{cv}(0) - 1800\ \text{lb}$$

$$= 3000\ \text{lb} - 1800\ \text{lb} = 1200\ \text{lb}$$

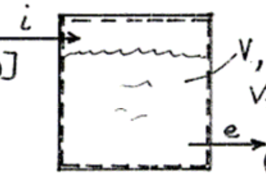
PROBLEM 4.7

KNOWN: A water storage tank contains a known volume of water. The volume flow rates in and out of the tank are given.

FIND: Determine how many days the tank will contain water.

SCHEMATIC & GIVEN DATA:

$(AV)_i = 5000[\exp(-t/20)]$
(t in days)



$V_1 = 100{,}000$ gal
$V_2 = 0$
$(AV)_e = 10{,}000$ gal/day

ENGR. MODEL: (1) The control volume is as shown on the above sketch. (2) The water is modeled as incompressible.

ANALYSIS: The mass rate balance reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e$$

With $m_{cv} = \rho V$, and $\dot{m}_i = \rho(AV)_i$ and $\dot{m}_e = \rho(AV)_e$

$$\cancel{\rho}\frac{dV}{dt} = \cancel{\rho}(AV)_i - \cancel{\rho}(AV)_e$$

$$\Rightarrow \quad \frac{dV}{dt} = (AV)_i - (AV_e) = 5000[\exp(-t/20)] - 10{,}000$$

Integrating from $t=0$ to $t=t_f$

$$V_2 - V_1 = \int_0^{t_f} (5000[\exp(-t/20)] - 10{,}000)\,dt$$

$$= \left(\frac{5000}{(-1/20)}\exp(-t/20) - 10{,}000t\right)\Bigg|_0^{t_f}$$

① $$= [-100{,}000\,\exp(-t_f/20) - 1] - 10{,}000\,t_f \qquad (*)$$

With $V_1 = 100{,}000$ gal and for $V_2 = 0$

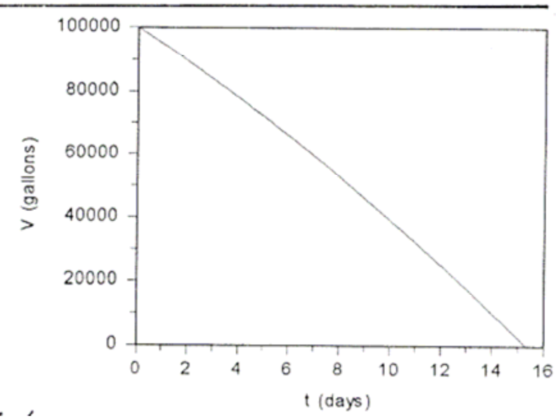
$$0 = 200{,}000 - 100{,}000\,\exp(-t_f/20) - 10{,}000\,t_f$$

Using IT to solve we get:

IT solution:
0 = 200000 - 100000 * exp(-tf/20) - 10000 * tf
// tf = 15.36 days ← t_f

1. To obtain V vs. t using *IT*:

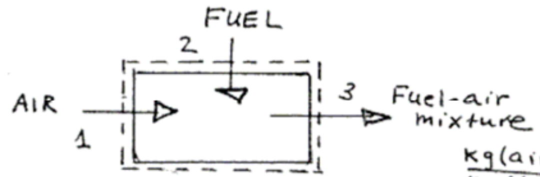
```
der(V,t) = AV1 - AV2
AV1 = 5000 * (exp(-t / 20))
AV2 = 10000
// Use the Explore button to sweep t from
// 0 to 15.36 days in steps of 0.05.
```



PROBLEM 4.8

A carburetor in an internal combustion engine mixes air with fuel to achieve a combustible mixture in which the *air-to-fuel ratio* is 20 kg (air) per kg (fuel). For a fuel mass flow rate of 5×10^{-3} kg/s, determine the mass flow rate of the mixture, in kg/s.

SCHEMATIC & GIVEN DATA:



$$\frac{kg(air)}{kg(fuel)} = 20$$

$$\dot{m}_2 = 5 \times 10^{-3}\ kg(fuel)/s.$$

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. All injected fuel exits at 3, mixed with air.

ANALYSIS:

At steady state, for each substance individually it is necessary for the incoming and outgoing mass flow rates to be equal. That is,

AIR: $\dot{m}_1 = \dot{m}_{A3}$

FUEL: $\dot{m}_2 = \dot{m}_{F3}$

We know that

$$\frac{\dot{m}_{A3}}{\dot{m}_{F3}} = 20\ \frac{kg(air)}{kg(fuel)}$$

$$\Rightarrow \dot{m}_{A3} = 20\ \frac{kg(air)}{kg(fuel)}\ \underbrace{\dot{m}_{F3}}_{=\dot{m}_2} = 20\ \frac{kg(air)}{kg(fuel)}\left(5 \times 10^{-3}\ \frac{kg(fuel)}{s}\right)$$

$$= 0.10\ \frac{kg(air)}{s}$$

The mass flow rate of the mixture is then

$$\dot{m}_3 = \dot{m}_{A3} + \dot{m}_{F3}$$

$$= 0.10\ \frac{kg(air)}{s} + 0.005\ \frac{kg(fuel)}{s}$$

$$= 0.105\ \frac{kg(mixture)}{s}$$

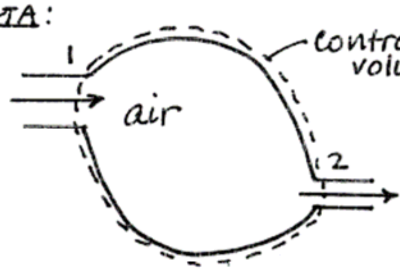
PROBLEM 4.9

KNOWN: Air flows through a one-inlet, one-exit control volume. Data are known at the inlet and exit.

FIND: Determine (a) the mass flow rate, (b) the exit area.

SCHEMATIC & GIVEN DATA:

$P_1 = 8$ bar
$T_1 = 600$ K
$V_1 = 40$ m/s
$A_1 = 20$ cm^2
$= 0.002$ m^2



$P_2 = 2$ bar
$T_2 = 400$ K
$V_2 = 350$ m/s

ENGR. MODEL: (1) The control volume is at steady state. (2) The air behaves as an ideal gas. (3) The flow is one-dimensional at the inlet and exit.

ANALYSIS: Beginning with the mass rate balance

$$\cancel{\frac{dm_{cv}}{dt}}^{0} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

Using data at the inlet and the ideal gas equation of state

(a) $$\dot{m} = \rho_1 A_1 V_1 = \left(\frac{P_1}{RT_1}\right) A_1 V_1$$

$$= \frac{(8 \text{ bar})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(600 \text{ K})} \left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right| (0.002 \text{ m}^2)(40 \text{ m/s})$$

$$= 0.3717 \text{ kg/s} \longleftarrow \dot{m}$$

(b) From $\dot{m}_1 = \dot{m}_2$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Thus

$$A_2 = \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{V_1}{V_2}\right) A_1$$

With $\rho = P/RT$

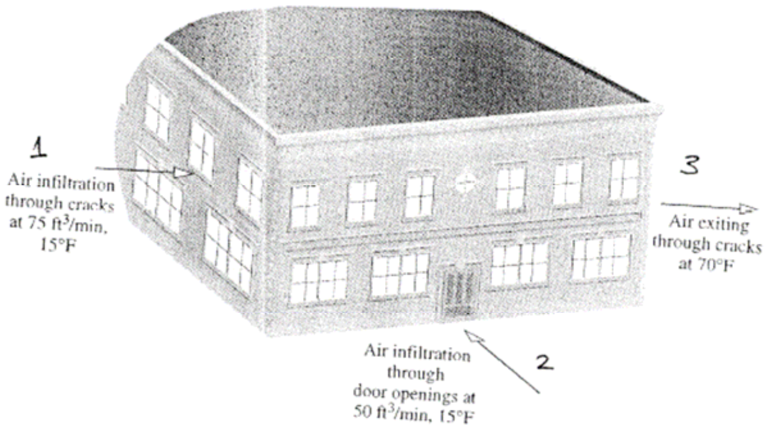
$$A_2 = \left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right) A_1$$

$$= \left(\frac{8}{2}\right)\left(\frac{400}{600}\right)\left(\frac{40}{350}\right)(20 \text{ cm}^2)$$

$$= 6.095 \text{ cm}^2 \longleftarrow A_2$$

PROBLEM 4.10

The small two-story office building shown in Fig. P4.10 has 36,000 ft^3 of occupied space. Due to cracks around windows and outside doors, air leaks in on the windward side of the building and leaks out on the leeward side of the building. Outside air also enters the building when outer doors are opened. On a particular day, tests were conducted. The outdoor temperature was measured to be 15°F. The inside temperature was controlled at 70°F. Keeping the doors closed, the infiltration rate through the cracks was determined to be 75 ft^3/min. The infiltration rate associated with door openings, averaged over the work day, was 50 ft^3/min. The pressure difference was negligible between the inside and outside of the building. (a) Assuming ideal gas behavior, determine at steady state the volumetric flow rate of air exiting the building, in ft^3/min. (b) When expressed in terms of the volume of the occupied space, determine the number of building air changes per hour.



ENGR. MODEL:

1. A control volume encloses the building.
2. The control volume is at steady state.
3. The air is modeled as an ideal gas.
4. Pressure is constant.

ANALYSIS: (a) At steady state $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$, where

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{(AV)_1}{(RT_1/p)}, \quad \dot{m}_2 = \frac{(AV)_2}{v_2} = \frac{(AV)_2}{RT_2/p}, \quad \dot{m}_3 = \frac{(AV)_3}{v_3} = \frac{(AV)_3}{RT_3/p}$$

Accordingly,

$$\frac{p(AV)_3}{RT_3} = \frac{p(AV)_1}{RT_1} + \frac{p(AV)_2}{RT_2} \Rightarrow (AV)_3 = \frac{T_3}{T_0}\left[(AV)_1 + (AV)_2\right]$$

T_1, T_2: equal ($= T_0$)

$$= \frac{530°R}{475°R}\left[75\,\frac{ft^3}{min} + 50\,\frac{ft^3}{min}\right]$$

$$= 139.5\,\frac{ft^3}{min} \quad \longleftarrow (a)$$

(b)

$$\begin{bmatrix}\text{Building Air}\\ \text{Changes}\\ \text{per hour}\end{bmatrix} = \frac{139.5\,\frac{ft^3}{min}\left|\frac{60\,min}{1\,h}\right|}{36{,}000\ ft^3/\text{airchange}}$$

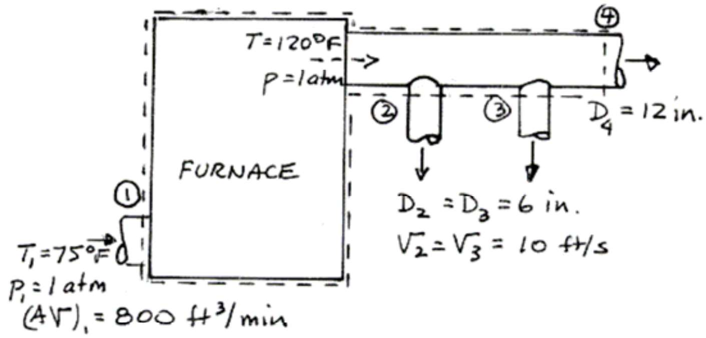
$$= 0.23 \text{ airchanges}/h \quad \longleftarrow (b)$$

PROBLEM 4.11:

KNOWN: Air enters a furnace operating at steady state and is to a duct system consisting of three ducts. Data are known at the inlet and each of the discharge ducts.

FIND: Determine (a) the mass flow rate entering the furnace, (2) the volumetric flow rate in each of the 6-in. exit ducts, (c) the velocity in the 12-in. exit duct.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control shown on the accompanying sketch is at steady state. (2) The air behaves as an ideal gas. (3) The temperature and pressure in each duct are the same as the temperature and pressure of the air delivered to the duct system.

ANALYSIS: (a) Using Eq. 4.4b with the ideal gas equation

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1}$$

$$= \frac{(1\text{ atm})(800\text{ ft}^3/\text{min})}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(535\,^\circ\text{R})}\left|\frac{14.696\text{ lbf/in.}^2}{1\text{ atm}}\right|\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right|\left|\frac{1\text{ min}}{60\text{ s}}\right|$$

$$= 0.989\text{ lb/s} \longleftarrow \dot{m}_1$$

(b) Since $D_2 = D_3$ and $V_2 = V_3$,

$$(AV)_2 = (AV)_3 = \left(\frac{\pi D^2}{4}\right)V = \left(\frac{\pi\left(\frac{6}{12}\right)^2 \text{ft}^2}{4}\right)\left(10\ \frac{\text{ft}}{\text{s}}\right)\left|\frac{60\text{ s}}{1\text{ min}}\right|$$

$$= 117.8\text{ ft}^3/\text{min} \longleftarrow (AV)_2, (AV)_3$$

(c) Applying the mass balance to get $\dot{m}_4$

$$\cancel{\frac{dm_{cv}}{dt}}^{\,0} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 - \dot{m}_4 \Rightarrow \dot{m}_4 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \qquad (*)$$

where

$$\dot{m}_2 = \dot{m}_3 = \frac{(AV)}{v} = \frac{p(AV)}{RT} = \frac{(14.696)(117.8)}{\left(\frac{1545}{28.97}\right)(580)}\left|\frac{144}{60}\right| = 0.1343\text{ lb/s}$$

From (*)

$$\dot{m}_4 = 0.989 - 2(.1343) = 0.7204\text{ lb/s}$$

Finally, from $\dot{m}_4 = \frac{A_4 V_4}{v_4}$

$$V_4 = \frac{v_4 \dot{m}_4}{A_4} = \frac{(RT_4)(\dot{m}_4)}{(p_4)\left(\frac{\pi D_4^2}{4}\right)} = \frac{\left(\frac{1545}{28.97}\right)(580)(0.7204)}{(14.696)\,\frac{\pi(12)^2}{4}} = 13.41\text{ ft/s} \longleftarrow V_4$$

PROBLEM 4.12

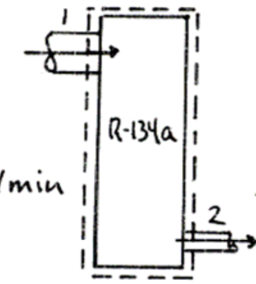
KNOWN: Refrigerant 134a flows through a refrigeration condenser. Data are known at the inlet and exit. The mass flow at the inlet is given.

FIND: Determine (a) the inlet velocity, (b) the diameter of the exit pipe.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The control volume is at steady state. (2) The flow is one-dimensional at the inlet and exit.

$P_1 = 9$ bar
$T_1 = 50\ ^\circ C$
$d_1 = 2.5$ cm
$\dot{m}_1 = 6$ kg/min



$P_2 = 9$ bar
$T_2 = 30\ ^\circ C$
$V_2 = 2.5$ m/s

ANALYSIS: (a) Solving Eq. 4.4b for V_1

$$V_1 = \frac{\dot{m}_1 v_1}{A_1} = \frac{\dot{m}_1 v_1}{\left(\frac{\pi d_1^2}{4}\right)} = \frac{4\dot{m}_1 v_1}{\pi d_1^2}$$

From Table A-12, $v_1 = 0.02472\ m^3/kg$. Thus

$$V_1 = \frac{(4)(6\ kg/min)(0.02472\ m^3/kg)}{\pi\ (2.5)^2 cm^2}\left|\frac{1\ min}{60\ s}\right|\left|\frac{10^4\ cm^2}{1\ m^2}\right|$$

$$= 5.04\ m/s \longleftarrow V_2$$

(b) To find the diameter of the exit pipe, begin with mass rate balance

$$\cancel{\frac{dm_{cv}}{dt}}^{\,0} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_2 = \dot{m}_1$$

Thus, with $\dot{m}_2 = A_2 V_2 / v_2$

$$A_2 = \frac{\dot{m}_2 v_2}{V_2}$$

From Table A-11, $T_2 < T_{sat\ @\ 9\ bar}$. Thus, the refrigerant is a sub-cooled liquid. From Table A-10,

$$v_2 \approx v_{f\ @\ 30\ ^\circ C} = 0.8417 \times 10^{-3}\ m^3/kg$$

Inserting values

$$A_2 = \frac{(6\ kg/min)(0.8417 \times 10^{-3}\ m^3/kg)}{(2.5\ m/s)}\left|\frac{1\ min}{60\ s}\right| = 3.367 \times 10^{-5}\ m^2$$

Finally, with $A = \pi d^2/4$

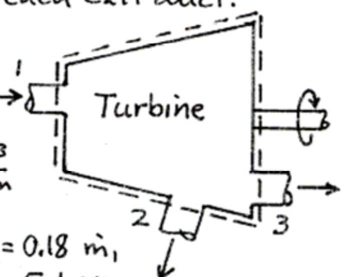
$$d_2 = \left(\frac{4A_2}{\pi}\right)^{1/2} = 0.0065\ m = 0.65\ cm \longleftarrow d_2$$

PROBLEM 4.13

KNOWN: Data are given for steam flowing through a turbine with one inlet and two exits.

FIND: Determine the diameter of each exit duct.

SCHEMATIC & GIVEN DATA:

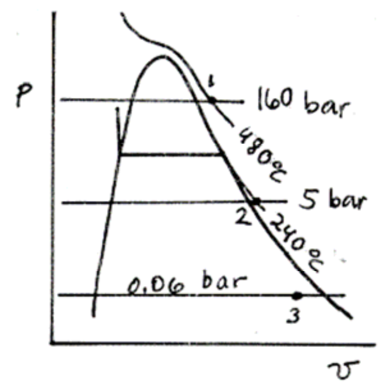


$P_1 = 160$ bar
$T_1 = 480$ °C
$(AV)_1 = 800 \frac{m^3}{min}$

$P_3 = 0.06$ bar
$x_3 = 0.94$
$V_3 = 400$ m/s

$\dot{m}_2 = 0.18\, \dot{m}_1$
$P_2 = 5$ bar
$T_2 = 240$ °C
$V_2 = 25$ m/s

ENGR. MODEL: (1) The control volume is at steady state. (2) The flow at the inlet and each exit is one-dimensional.



ANALYSIS: The mass flow rate at the inlet is

$$\dot{m}_1 = \frac{(AV)_1}{v_1}$$

From Table A-4, $v_1 = 0.01842$ m³/kg. Thus

$$\dot{m}_1 = \frac{(800\ m^3/min)}{(0.01842\ m^3/kg)} \left| \frac{1\ min}{60\ s} \right| = 723.9\ kg/s$$

and $\dot{m}_2 = 0.18\, \dot{m}_1 = 130.3$ kg/s

Applying the mass rate balance

$$\cancel{\frac{dm_{cv}}{dt}}^{0} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 593.6\ kg/s$$

Now, with $\dot{m} = AV/v$, and from Table A-4; $v_2 = 0.4646$ m³/kg

$$A_2 = \frac{\dot{m}_2 v_2}{V_2} = \frac{(130.3\ kg/s)(0.4646\ m^3/kg)}{(25\ m/s)} = 2.421\ m^2$$

Noting that $A = \pi d^2/4$

$$d_2 = \sqrt{\frac{4 A_2}{\pi}} = 1.756\ m \longleftarrow d_2$$

From Table A-3, at $P_3 = 0.06$ bar and $x_3 = 0.94$

$$v_3 = v_{f3} + x_3 (v_{g3} - v_{f3})$$
$$= 1.0064 \times 10^{-3} + (0.94)(23.739 - 1.0064 \times 10^{-3}) = 22.315\ m^3/kg$$

Thus

$$A_3 = \frac{\dot{m}_3 v_3}{V_3} = \frac{(593.6)(22.315)}{(400)} = 33.115\ m^2$$

and

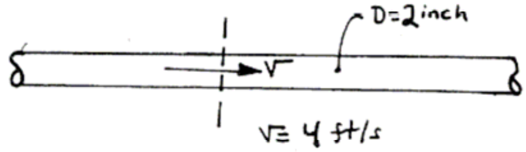
$$d_3 = \sqrt{\frac{4 A_3}{\pi}} = 6.49\ m \longleftarrow d_3$$

PROBLEM 4.14

KNOWN: Data are provided for substances flowing through a pipe of known diameter.

FIND: Determine the mass flow rate for each of three specified substances.

SCHEMATIC & GIVEN DATA:



ANALYSIS: With Eq. 4.4b

$$\dot{m} = \frac{AV}{v} = \frac{(\pi D^2/4)V}{v} = \frac{\pi\left[\left(\frac{1}{12}\text{ ft}\right)^2/4\right]\left[4\text{ ft/s}\right]}{v} = \frac{0.02182\text{ ft}^3/\text{s}}{v}$$

(a) water at 50 lbf/in², 80°F

with $v \approx v_f(80°F) = 0.01607\text{ ft}^3/\text{lb}$ (Table A-2E)

$$\therefore \dot{m} = \frac{0.02182\text{ ft}^3/\text{s}}{0.01607\text{ ft}^3/\text{lb}} = 1.35760\text{ lb/s} \longleftarrow \text{(a)}$$

(b) Nitrogen as an ideal gas at 50 lbf/in², 80°F = 540°R

$$v = \frac{RT}{p} = \frac{\left(\frac{1545}{28.01}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(540°\text{R})}{(50\text{ lbf/in}^2)}\left|\frac{1\text{ ft}^2}{144\text{ in}^2}\right| = 4.137\text{ ft}^3/\text{lb}$$

$$\therefore \dot{m} = \frac{0.02182\text{ ft}^3/\text{s}}{4.137\text{ ft}^3/\text{lb}} = 0.00527\text{ lb/s} \longleftarrow \text{(b)}$$

(c) R22 at 50 lbf/in², 80°F

$v = 1.2716\text{ ft}^3/\text{lb}$ from Table A-9E

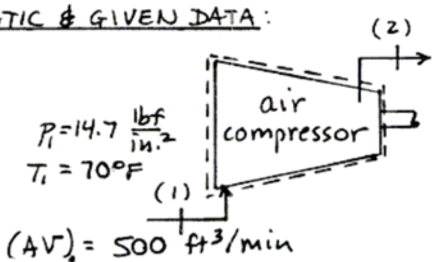
$$\therefore \dot{m} = \frac{0.02182\text{ ft}^3/\text{s}}{1.2716\text{ ft}^3/\text{lb}} = 0.01716\text{ lb/s} \longleftarrow \text{(c)}$$

PROBLEM 4.15

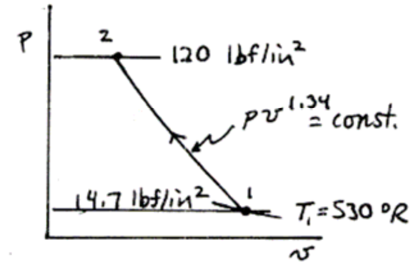
KNOWN: Data are known at the inlet and exit of an air compressor operating at steady state. Each unit of mass passing through undergoes a polytropic process.

FIND: Determine the temperature and diameter at the exit.

SCHEMATIC & GIVEN DATA:



$V_2 = 700\ ft/s$
$P_2 = 120\ lbf/in^2$



ENGR. MODEL: (1) The control volume is at steady state. (2) Each unit of mass undergoes a process described by $pv^{1.34} = $ constant. (3) The air behaves as an ideal gas, as can be verified by reference to the compressibility chart.

ANALYSIS: The mass rate balance reads,

$$\cancel{\frac{dm_{cv}}{dt}}^{0} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \Rightarrow \frac{(AV)_1}{v_1} = \frac{(AV)_2}{v_2} \qquad (*)$$

Evaluate v_1 using the ideal gas equation

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{1545}{28.97}\frac{ft\cdot lbf}{lb\cdot °R}\right)(530\ °R)}{(14.7\ lbf/in^2)}\left|\frac{1\ ft^2}{144\ in^2}\right| = 13.35\ ft^3/lb$$

With assumption (2),

$$v_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1.34}} v_1 = \left(\frac{14.7}{100}\right)^{\frac{1}{1.34}}\left(13.35\ \frac{ft^3}{lb}\right) = 3.192\ ft^3/lb$$

The exit temperature is determined using the ideal gas equation of state.

$$T_2 = \frac{P_2 v_2}{R} = \frac{(120\ lbf/in^2)(3.192\ ft^3/lb)}{\left(\frac{1545}{28.97}\ \frac{ft\ lbf}{lb\cdot °R}\right)}\left|\frac{144\ in^2}{1\ ft^2}\right|$$

$$= 1034\ °R = 574\ °F \longleftarrow T_2$$

Solving (*) for A_2 and inserting values

$$A_2 = \frac{(AV)_1\ v_2}{V_2\ v_1}$$

$$= \frac{(500\ ft^3/min)(3.192\ ft^3/lb)}{(700\ ft/s)(13.35\ ft^3/lb)}\left|\frac{1\ min}{60\ s}\right| = 0.002846\ ft^2$$

$$= 0.410\ in^2$$

Using $A_2 = \pi d_2^2/4$

$$d_2 = \left(\frac{4A_2}{\pi}\right)^{1/2} = \left(\frac{(4)(0.410\ in^2)}{\pi}\right)^{1/2}$$

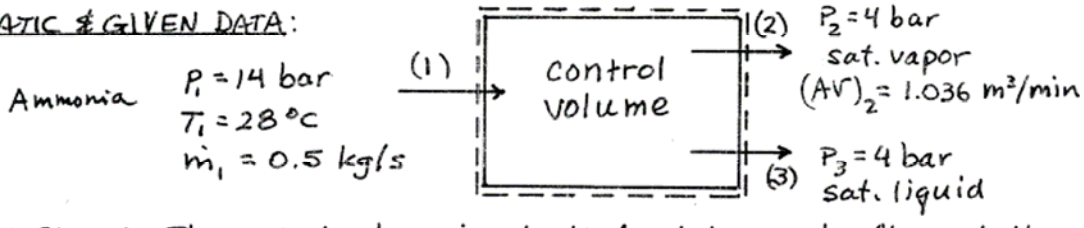
$$d_2 = 0.722\ in \longleftarrow d_2$$

PROBLEM 4.16

KNOWN: Ammonia flows through a control volume at steady state. The control volume has one inlet and two exit, and data are known at each flow boundary.

FIND: Determine (a) the minimum inlet diameter so the ammonia velocity does not exceed 20 m/s. (b) the volumetric flow rate of the second exit stream.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) The flow at the inlet is one-dimensional.

ANALYSIS: (a) To relate velocity and pipe diameter at the inlet, use Eq. 4.4b

$$V_1 = \frac{\dot{m}_1 v_1}{A_1} = \frac{\dot{m}_1 v_1}{\left(\frac{\pi d_1^2}{4}\right)}$$

Thus, velocity varies inversely with diameter. The minimum diameter corresponds to $V_1 = 20$ m/s.

To get v_1, note from Table A-14 that $T_1 = 28°C$ is less than T_{sat} at 14 bar. Hence, from Table A-13, $v_1 \approx v_{f@28°C} = 1.6714 \times 10^{-3}\ m^3/kg$, and

$$(d_1)_{min} = \sqrt{\frac{4\ \dot{m}_1 v_1}{\pi V_1}} = \sqrt{\frac{(4)(0.5\ kg/s)(1.6714 \times 10^{-3}\ m^3/kg)}{\pi\ (20\ m/s)}}$$

$$= 0.00729\ m = 0.729\ cm \longleftarrow (d_1)_{min}$$

(b) To find $(AV)_3$, begin with the mass rate balance

$$\cancel{\frac{dm_{cv}}{dt}}^{0} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2$$

With $\dot{m} = (AV)/v$

$$(AV)_3 = v_3 \left[\dot{m}_1 - (AV)_2 / v_2 \right]$$

From Table A-14 at 4 bar ; $v_2 = 0.3094\ m^3/kg$ and $v_3 = 1.5597 \times 10^{-3}\ m^3/kg$. Thus

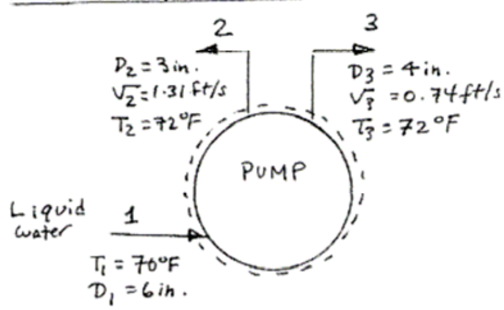
$$(AV)_3 = \left(1.5597 \times 10^{-3} \frac{m^3}{kg}\right) \left[\left(0.5 \frac{kg}{s}\right)\left(\frac{60\ s}{1\ min}\right) - \frac{(1.036\ m^3/min)}{(0.3094\ m^3/kg)} \right]$$

$$= 0.0416\ m^3/min \longleftarrow (AV)_3$$

PROBLEM 4.17

Liquid water at 70°F enters a pump though an inlet pipe having a diameter of 6 in. The pump operates at steady state and supplies water to two exit pipes having diameters of 3 in. and 4 in., respectively. The velocity of the water exiting the 3-in. pipe is 1.31 ft/s. At the exit of the 4-in. pipe the velocity is 0.74 ft/s. The temperature of the water in each exit pipe is 72°F. Determine (a) the mass flow rate, in lb/s, in the inlet pipe and each of the exit pipes, and (b) the volumetric flow rate at the inlet, in ft³/min.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The control volume shown in the sketch is at steady state.
2. For the liquid water, $v \approx v_f(T)$.

ANALYSIS: (a) The mass flow rates at 2 and 3 are given by $\dot{m} = \frac{AV}{v}$. That is, with assumption (2)

$$\dot{m}_2 = \frac{A_2 V_2}{v_f(T_2)} = \frac{\left(\frac{\pi (3/12)^2 \text{ ft}^2}{4}\right)(1.31 \text{ ft/s})}{(0.01606 \text{ ft}^3/\text{lb})} = 4 \text{ lb/s}$$

$$\dot{m}_3 = \frac{A_3 V_3}{v_f(T_3)} = \frac{\left(\frac{\pi (4/12)^2 \text{ ft}^2}{4}\right)(0.74 \text{ ft/s})}{(0.01606 \text{ ft}^3/\text{lb})} = 4.02 \text{ lb/s}$$

(a)

A mass rate balance gives at steady state,

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 = 4 \frac{\text{lb}}{\text{s}} + 4.02 \frac{\text{lb}}{\text{s}} = 8.02 \frac{\text{lb}}{\text{s}}.$$

(b) The volumetric flow rate at 1 is

$$(AV)_1 = \dot{m}_1 v_f(T_1)$$

$$= \left(8.02 \frac{\text{lb}}{\text{s}}\right)\left(0.01605 \frac{\text{ft}^3}{\text{lb}}\right)\left|\frac{60 \text{ s}}{1 \text{ min}}\right|$$

$$= 7.72 \text{ ft}^3/\text{min}$$

(b)

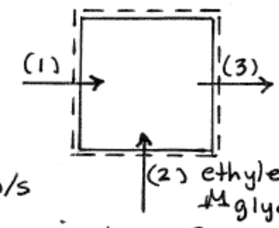
PROBLEM 4.18

KNOWN: Separate streams of water and ethylene glycol (glycol) mix to form a single mixture that is half glycol by mass. Data are given for the inlet flows.

FIND: Determine (a) the molar and volumetric flow rates of the entering glycol. (b) the diameters of the supply pipes.

SCHEMATIC & GIVEN DATA:

Water:
$T_1 = 20°C$
$p_1 = 1$ bar
$\dot{n}_1 = 4.2 \frac{kmol}{min}$
$V_1 = V_2 = 2.5$ m/s



Mixture: 50% glycol by mass

(2) ethylene glycol
$M_{glycol} = 62.07$ kg/kmol
$\rho_{glycol} = 1.115\, \rho_{H_2O}$

ENGR. MODEL: (1) The control volume is at steady state. (2) The water and glycol are each incompressible substances.

ANALYSIS: (a) To find the glycol flow rates, begin with a mass rate balance. At steady state: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$. For the mixture

$$\left.\begin{aligned} 50\%\text{ glycol} &\Rightarrow \frac{\dot{m}_2}{\dot{m}_3} = 0.5 \\ 50\%\text{ water} &\Rightarrow \frac{\dot{m}_1}{\dot{m}_3} = 0.5 \end{aligned}\right\} \Rightarrow \dot{m}_1 = \dot{m}_2.$$

Now

$$\dot{m}_1 = \dot{n}_1 M_{H_2O} = \left(4.2 \frac{kmol}{min}\right)\left(18.02 \frac{kg}{kmol}\right) = 75.68 \text{ kg/min}$$

$$\dot{n}_2 = \frac{\dot{m}_2}{M_{glycol}} = \frac{75.68 \text{ kg/min}}{62.07 \text{ kg/kmol}} = 1.219 \text{ kmol/min} \longleftarrow \dot{n}_2$$

Also, with $\dot{m} = \rho(AV)$

$$(AV)_2 = \frac{\dot{m}_2}{\rho_{glycol}} = \frac{\dot{m}_2}{1.115\, \rho_{H_2O}}$$

From Table A-2, $v_{H_2O} \approx v_f @ 20°C = 1.0018 \times 10^{-3}$ m³/kg and $\rho_{H_2O} = 1/v_{H_2O} = 998.2$ kg/m³. Thus

$$(AV)_2 = \frac{75.68 \text{ kg/min}}{1.115\,(998.2) \text{ kg/m}^3} = 0.068 \text{ m}^3\text{/min} \longleftarrow (AV)_2$$

(b) The area of the water supply pipe is

$$A_1 = \frac{\dot{m}_1}{\rho_{H_2O} V_1} = \frac{(75.68 \text{ kg/min})}{(998.2 \text{ kg/m}^3)(2.5 \text{ m/s})} \left|\frac{1 \text{ min}}{60 \text{ s}}\right| \left|\frac{10^4 \text{ cm}^2}{1 \text{ m}^2}\right| = 5.054 \text{ cm}^2$$

With $A = \pi d^2/4$

$$d_1 = \sqrt{\frac{4A_1}{\pi}} = \sqrt{\frac{(4)(5.054 \text{ cm}^2)}{\pi}} = 2.54 \text{ cm} \longleftarrow d_1$$

Similarly for the glycol stream

$$A_2 = \frac{75.68}{(1.115)(998.2)(2.5)} \left|\frac{10^4}{60}\right| = 4.533 \text{ cm}^2$$

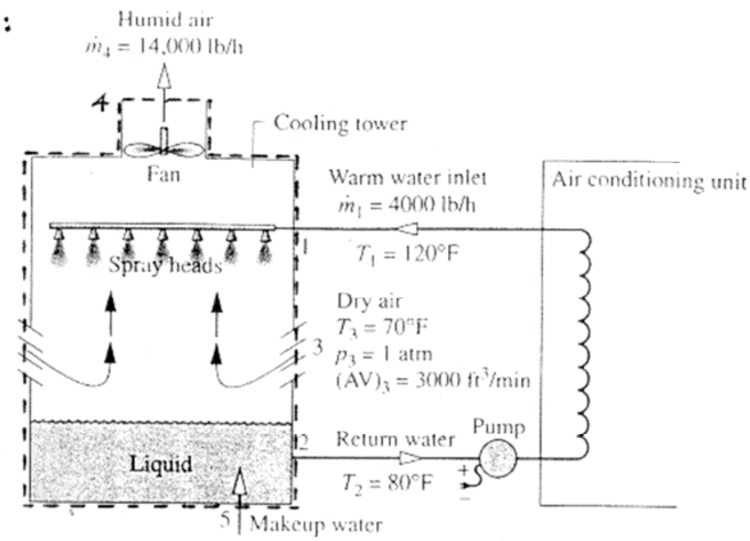
$$d_2 = \sqrt{\frac{(4)(4.533)}{\pi}} = 2.4 \text{ cm} \longleftarrow d_2$$

PROBLEM 4.19

KNOWN: Data are know for inlet and exit streams of an air conditioner cooling tower operating at steady state.

FIND: Determine the mass flow rate of the makeup water.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) The dry air stream at location 3 behaves as an ideal gas. (This can be checked using the compressibility chart.)

ANALYSIS: The mass rate balance for the control reduces as follows:

$$\cancel{\frac{dm_{cv}}{dt}}^{\,0} = \dot{m}_1 + \dot{m}_3 + \dot{m}_5 - \dot{m}_2 - \dot{m}_4$$

$$\therefore \quad \dot{m}_5 = \dot{m}_2 - \dot{m}_1 + \dot{m}_4 - \dot{m}_3$$

From the schematic, we see that circuit through the air conditioning unit is closed. Therefore, at steady state $\dot{m}_1 = \dot{m}_2$. Thus

$$\dot{m}_5 = \dot{m}_4 - \dot{m}_3 \qquad (*)$$

To get $\dot{m}_3$, use Eq. 4.4b and the ideal gas equation

$$\dot{m}_3 = \frac{(AV)_3}{v_3} = \frac{p_3 (AV)_3}{R T_3}$$

$$= \frac{(1\text{ atm})(3000\text{ ft}^3/\text{min})}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(530°\text{R})} \left|\frac{14.696\text{ lbf/in}^2}{1\text{ atm}}\right| \left|\frac{144\text{ in}^2}{1\text{ ft}^2}\right| \left|\frac{60\text{ min}}{1\text{ h}}\right|$$

$$= 13480 \text{ lb/h}$$

Finally, from (*)

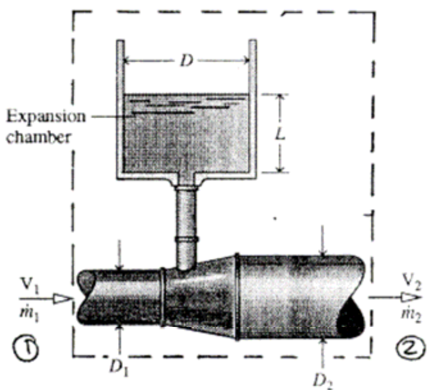
$$\dot{m}_5 = 14,000 - 13,480 = 520 \text{ lb/h} \longleftarrow \dot{m}_5$$

PROBLEM 4.20

KNOWN: A pipe carrying an incompressible liquid contains an expansion chamber.

FIND: (a) Develop an expression for the rate of change of liquid level in the chamber in terms of certain quantities. (b) Compare the relative magnitudes of the mass flow rates for specified values for dL/dt, the rate of change of liquid level.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume is shown on the accompanying schematic. 2. The liquid is modeled as incompressible. 3. Flow is one-dimensional at 1, 2.

ANALYSIS: (a) The mass rate balance for the control volume is

$$\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2$$

with

$$m_{cv} = \rho\left(\frac{\pi D^2}{4}\right)L$$

$$\dot{m}_1 = \rho\left(\frac{\pi D_1^2}{4}\right)V_1$$

$$\dot{m}_2 = \rho\left(\frac{\pi D_2^2}{4}\right)V_2$$

the mass rate balance becomes

$$\rho\left(\frac{\pi D^2}{4}\right)\frac{dL}{dt} = \rho\left(\frac{\pi D_1^2}{4}\right)V_1 - \rho\left(\frac{\pi D_2^2}{4}\right)V_2$$

or, solving for dL/dt and simpifying

$$\frac{dL}{dt} = \frac{D_1^2 V_1 - D_2^2 V_2}{D^2} \longleftarrow dL/dt$$

(b) The mass flow rate expressions indicate $\dot{m}_1 \sim D_1^2 V_1$, $\dot{m}_2 \sim D_2^2 V_2$.

Thus,

$$\frac{dL}{dt} > 0 \Rightarrow D_1^2 V_1 > D_2^2 V_2 \Rightarrow \dot{m}_1 > \dot{m}_2$$

$$\frac{dL}{dt} = 0 \Rightarrow D_1^2 V_1 = D_2^2 V_2 \Rightarrow \dot{m}_1 = \dot{m}_2$$

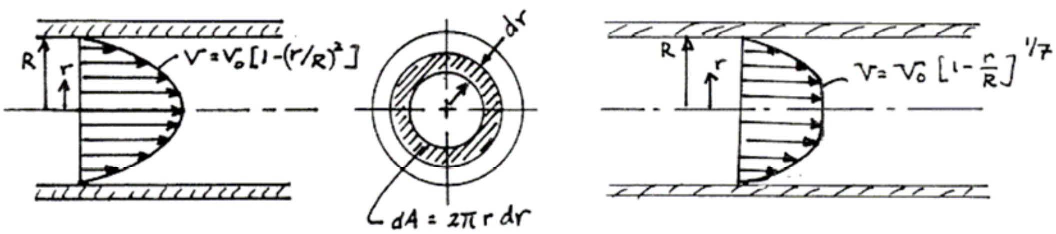
$$\frac{dL}{dt} < 0 \Rightarrow D_1^2 V_1 < D_2^2 V_2 \Rightarrow \dot{m}_1 < \dot{m}_2$$

PROBLEM 4.21

KNOWN: Velocity distributions are given for laminar and turbulent flow of an incompressible liquid in a circular pipe.

FIND: For each distribution, plot V/V_0 vs. r/R, derive expressions for the mass flow rate, average flow velocity, and specific kinetic energy. Determine the percent error if the specific kinetic energy is evaluated in terms of average velocity. Discuss.

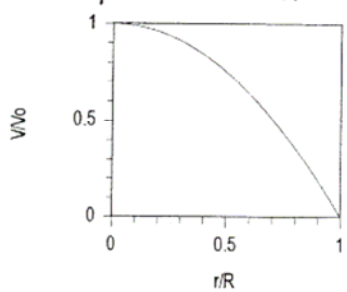
SCHEMATIC & GIVEN DATA:



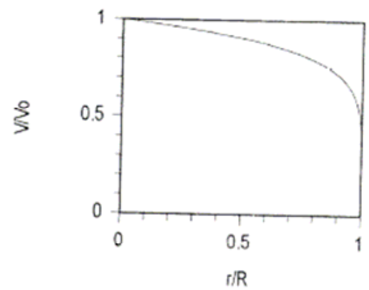
LAMINAR TURBULENT

ENGR. MODEL: The liquid is modeled as incompressible.

ANALYSIS: (a)



Laminar



Turbulent

(b) Using Eq. 4.3, the mass flow rate for laminar flow is

$$\dot{m} = \int_A \rho V\,dA$$

$$= \int_0^R \rho V_0\left[1-\left(\frac{r}{R}\right)^2\right]2\pi r\,dr$$

$$= \rho V_0 2\pi \int_0^R \left[r - \frac{r^3}{R^2}\right]dr$$

$$= \rho V_0 2\pi \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]\Bigg|_{r=0}^{r=R} = \rho V_0 2\pi\left[\frac{R^2}{2} - \frac{R^2}{4}\right]$$

$$= \rho V_0 \pi \frac{R^2}{2}$$

Using Eq. 4.4a, the average velocity is

$$V_{ave} = \frac{\dot{m}}{\rho A} = \frac{\rho V_0 \pi R^2/2}{\rho(\pi R^2)}$$

$$= \frac{V_0}{2}$$

Continued on next slide

Problem 4-21 continued

Using Eq. 4.3, the mass flow rate for turbulent flow is

$$\dot{m} = \int_A \rho V \, dA$$

$$= \int_0^R \rho V_0 \left[1 - \frac{r}{R}\right]^{\frac{1}{7}} 2\pi r \, dr$$

$$= \rho V_0 2\pi \int_0^R r \left[1 - \frac{r}{R}\right]^{\frac{1}{7}} dr$$

To evaluate the integral, let $u = 1 - r/R$ and $dr = -R\,du$. Then

$$\dot{m} = \rho V_0 2\pi \int_1^0 R(1-u)\, u^{1/7} (-R\,du) = \rho V_0 2\pi \left[-R^2 \int_1^0 (u^{1/7} - u^{8/7})\, du\right]$$

$$= \rho V_0 2\pi \left[-R^2 \left(\frac{7}{8} u^{8/7} - \frac{7}{15} u^{15/7}\right)\Bigg|_{u=1}^{u=0}\right]$$

$$= \rho V_0 2\pi \left[R^2 \frac{49}{120}\right] = \rho V_0 \pi \left(\frac{49}{60}\right) R^2$$

Using Eq. 4.4a, the average velocity is

$$V_{ave} = \frac{\dot{m}}{\rho A} = \frac{\rho V_0 \pi \left(\frac{49}{60}\right) R^2}{\rho (\pi R^2)} = \frac{49}{60} V_0$$

(c) The specific kinetic energy (kinetic energy per unit of mass) carried through an area normal to the flow is

$$ke = \frac{\int_A \frac{V^2}{2} \rho V \, dA}{\dot{m}} = \frac{\rho \int_A \frac{V^3}{2} dA}{\rho \int_A V \, dA} = \frac{\int_A \frac{1}{2} V^3 dA}{\int_A V \, dA} = \frac{\int_A \frac{1}{2} V^3 dA}{V_{ave} A} \qquad (*)$$

Forming the ratio of Eq. (*) to the specific kinetic energy calculated as $V_{ave}^2/2$, we have the kinetic energy coefficient (or kinetic energy correction factor):

$$\alpha = \frac{\int_A \frac{1}{2} V^3 dA}{\frac{1}{2} V_{ave}^3 A} = \frac{\int_A V^3 dA}{V_{ave}^3 A}$$

For Laminar Flow: $\alpha = 2$ ➡ % ERROR = 50

① For Turbulent Flow: $\alpha = 1.058$ ➡ % ERROR = 5.5

The flatter turbulent velocity profile adheres most closely to the idealizations of one-dimensional flow.

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1. For further discussion, see R. W. Fox and A. T. McDonald, Introduction to Fluid Mechanics, 5th ed., J. Wiley & Sons, New York, pp 354-356.

PROBLEM 4.22

KNOWN: Data are provided for a cylindrical tank being drained of water.

FIND: Determine the time, in min., when the tank holds 900 kg of water.

SCHEMATIC & GIVEN DATA:

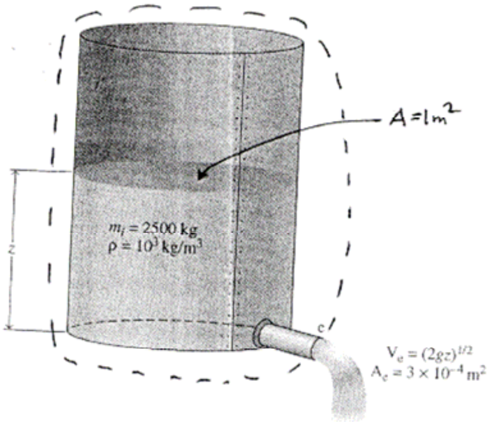


Fig. P4.22

ENGR. MODEL:

1. As shown in the sketch, a control volume enclosing the tank is being considered.
2. $g = 9.81\ m/s^2$
3. The density of water is $10^3\ kg/m^3$.

ANALYSIS: Mass rate balance: $\frac{dm_{cv}}{dt} = -\dot{m}_e$, where $\dot{m}_e$ is the mass flow rate at the exit. That is, $\dot{m}_e = \rho_e A_e V_e$ (Eq. 4.4a). Accordingly

constant at ρ

$$\frac{dm_{cv}}{dt} = -\rho A_e V_e \quad \text{, where } V_e = (2gz)^{1/2}$$

$$= -\rho A_e (2gz)^{1/2} \qquad (1)$$

Also, note that $m_{cv}(t) = \rho A z$, where A is the area of the water free surface. Thus, $z = m_{cv}(t)/\rho A$ and Eq. (1) becomes

$$\frac{dm_{cv}}{dt} = -\rho A_e \left[\frac{2g\, m_{cv}(t)}{\rho A}\right]^{1/2} = -\left[\frac{2g\rho A_e^2}{A}\right]^{\frac{1}{2}} (m_{cv}(t))^{1/2}$$

Thus,

$$\frac{1}{(m_{cv})^{1/2}} \frac{dm_{cv}}{dt} = -\left[\frac{2g\rho A_e^2}{A}\right]^{1/2}$$

Integrating from $t=0$ to t_f, we get $2(m_{cv})^{1/2}\Big]_0^{t_f} = -\left[\frac{2g\rho A_e^2}{A}\right]^{1/2} t_f$. Or

$$t_f = -\left[\frac{2A}{g\rho A_e^2}\right]^{1/2} \left[(m_{cv}(t_f))^{1/2} - (m_{cv}(0))^{1/2}\right] \qquad (2)$$

Inserting $A = 1\ m^2$, $g = 9.81\ m/s^2$, $\rho = 10^3\ kg/m^3$, $A_e = 3\times10^{-4}\ m^2$, $m_{cv}(0) = 2500\ kg$, and $m_{cv}(t_f) = 900\ kg$, we get

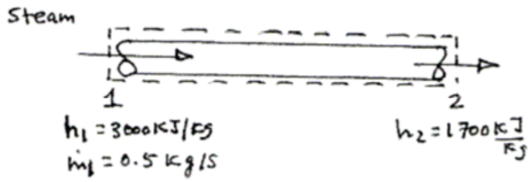
$$t_f = -\left[\frac{2(1\ m^2)}{(9.81\ \frac{m}{s^2})(10^3\frac{kg}{m^3})(3\times10^{-4}\ m^2)^2}\right]^{1/2} \left[(900\ kg)^{1/2} - (2500\ kg)^{1/2}\right] \left|\frac{1\ min}{60\ s}\right|$$

$$= 15.86\ min$$

PROBLEM 4.23

Steam enters a horizontal pipe operating at steady state with a specific enthalpy of 3000 kJ/kg and a mass flow rate of 0.5 kg/s. At the exit, the specific enthalpy is 1700 kJ/kg. If there is no significant change in kinetic energy from inlet to exit, determine the rate of heat transfer between the pipe and its surroundings, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} \equiv 0$, there is no significant change in kinetic energy from inlet to exit, and $\Delta pe = 0$ (horizontal tube).

ANALYSIS: Reducing Eq. 4.20a

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

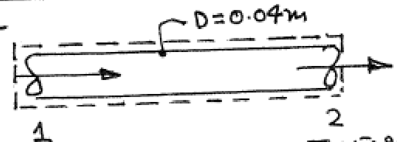
$$\Rightarrow \quad \dot{Q}_{cv} = \dot{m}\left[h_2 - h_1\right] = (0.5\ \text{kg/s})(1700 - 3000)\ \frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = -650\ \text{kW} \longleftarrow$$

PROBLEM 4.24

Refrigerant 134a enters a horizontal pipe operating at steady state at 40°C, 300 kPa and a velocity of 40 m/s. At the exit, the temperature is 50°C and the pressure is 240 kPa. The pipe diameter is 0.04 m. Determine (a) the mass flow rate of the refrigerant, in kg/s, (b) the velocity at the exit, in m/s, and (c) the rate of heat transfer between the pipe and its surroundings, in kW.

SCHEMATIC & GIVEN DATA:

R134a



$T_1 = 40°C$
$p_1 = 300 kPa$
$V_1 = 40 m/s$

$T_2 = 50°C$
$p_2 = 240 kPa$

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and $\Delta pe = 0$ (horizontal).

ANALYSIS: (a) Using Eq. 4.4a,

$$\dot{m}_1 = \frac{A V_1}{v_1} = \frac{\left(\frac{\pi (0.04\,m)^2}{4}\right)(40\,\frac{m}{s})}{0.08089\,\frac{m^3}{kg}}$$

Table A-12

$$\dot{m}_1 = 0.621\ kg/s \quad \longleftarrow (a)$$

(b) $\dot{m}_1 = \dot{m}_2$ (steady state)

$$\Rightarrow \frac{A V_1}{v_1} = \frac{A V_2}{v_2} \Rightarrow V_2 = \frac{v_2}{v_1} V_1$$

$$\therefore V_2 = \left(\frac{0.10562}{0.08089}\right)(40\,m/s) = 52.23\ m/s \quad \longleftarrow (b)$$

(c) Reducing Eq. 6.20a,

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g\cancel{(z_1 - z_2)}\right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}\right]$$

unit conversions on k.e. term

$$= 0.621\,\frac{kg}{s}\left[(294.47 - 284.05)\,\frac{kJ}{kg} + \left[\frac{(52.23)^2 - (40)^2}{2}\left(\frac{m^2}{s^2}\right)\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|\right]\right]$$

(Table A-12)

$$= 0.621\,\frac{kg}{s}\left[\ 10.42 + 0.56\ \right]\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right| = +6.82\ kW \quad \longleftarrow (c)$$

PROBLEM 4.25

As shown in Fig. P4.25, air enters a pipe at 25°C, 100 kPa with a volumetric flow rate of 23 m³/h. On the outer pipe surface is an electrical resistor covered with insulation. With a voltage of 120 V, the resistor draws a current of 4 amps. Assuming the ideal gas model with $c_p = 1.005$ kJ/kg · K for air and ignoring kinetic and potential energy effects, determine (a) the mass flow rate of the air, in kg/h, and (b) the temperature of the air at exit, in °C.

SCHEMATIC & GIVEN DATA:

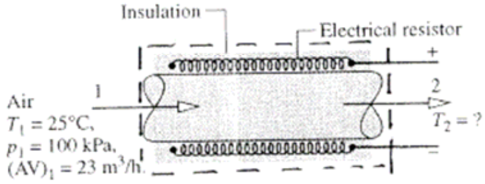


Fig. P4.25

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects can be ignored.
3. The air can be modeled as an ideal gas with constant specific heat cp.

ANALYSIS:

(a) $\dot{m} = \dfrac{(AV)_1}{v_1} = \dfrac{p_1 (AV)_1}{RT_1} = \dfrac{(10^5 \text{N/m}^2)(23 \text{ m}^3/\text{h})}{\left(\frac{8314}{28.97} \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(298\text{K})} = 26.89 \ \frac{\text{kg}}{\text{h}}$ ← (a)

(b) Reducing Eq. 4.20a using listed assumptions,

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

where $(h_1 - h_2) = c_{pa}(T_1 - T_2)$ and

$$\dot{W}_{cv} = -(\text{voltage})(\text{current})$$
$$= -(120\text{ volts})(4\text{ amps})\left|\frac{1 \text{ Watt/amp}}{1 \text{ Volt}}\right|\left|\frac{1 \text{ kW}}{10^3 \text{ Watt}}\right|$$
$$= -0.48 \text{ kW}$$

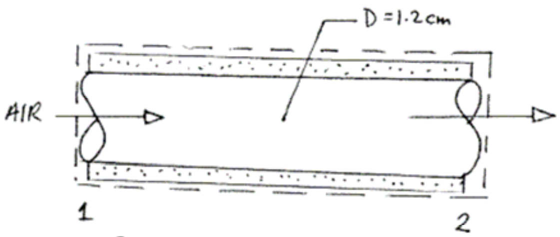
Collecting results, and solving for T_2,

$$T_2 = T_1 - \frac{\dot{W}_{cv}}{\dot{m}\, c_{pa}}$$
$$= 298 - \frac{(-0.48\text{kW})}{\left(26.89 \frac{\text{kg}}{\text{h}}\right)\left(1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\left|\frac{3600\text{s}}{1\text{h}}\right|\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right|$$
$$= 362\text{K}\ (89°\text{C})$$ ← (b)

PROBLEM 4.27

Air at 600 kPa, 330 K enters a well-insulated, horizontal pipe having a diameter of 1.2 cm and exits at 120 kPa, 300 K. Applying the ideal gas model for air, determine at steady state (a) the inlet and exit velocities, each in m/s, and (b) the mass flow rate, in kg/s.

SCHEMATIC & GIVEN DATA:



1: $T_1 = 330K$, $P_1 = 600 kPa$

2: $T_2 = 300K$, $P_2 = 120 kPa$

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer can be ignored. Also, $\Delta pe = 0$ (horizontal); $\dot{W}_{cv} = 0$.
3. The air can be modeled as an ideal gas

ANALYSIS: (a) Mass rate balance, $\dot{m}_1 = \dot{m}_2$. That is

$$\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2} \Rightarrow \quad V_1 = \frac{v_1}{v_2} V_2 = \frac{(RT_1/P_1)}{(RT_2/P_2)} V_2$$

$$V_1 = \left(\frac{P_2}{P_1}\right)\left(\frac{T_1}{T_2}\right) V_2 = \left(\frac{120 kPa}{600 kPa}\right)\left(\frac{330K}{300K}\right) V_2$$

$$V_1 = 0.22 V_2 \qquad (1)$$

Energy rate balance,

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g\cancel{(z_1 - z_2)}\right]$$

$$\Rightarrow \quad 0 = (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} \Rightarrow 0 = (h_1 - h_2) + \frac{(0.22 V_2)^2 - V_2^2}{2}$$

Or, with data from Table A-22 for h_1 and h_2,

$$\frac{0.9516 V_2^2}{2} = (330.34 - 300.19)\frac{kJ}{kg}$$

$$\Rightarrow \quad V_2 = \sqrt{\frac{2(330.34 - 300.19)}{0.9516} kJ/kg \left|\frac{1 kg \cdot m/s^2}{1 N}\right|\left|\frac{10^3 N \cdot m}{1 kJ}\right|}$$

$$= 251.73 \text{ m/s} \qquad (a)$$

And with Eq.(1)

$$V_1 = 0.22 V_2 = 55.38 \text{ m/s}$$

(b) Then, with

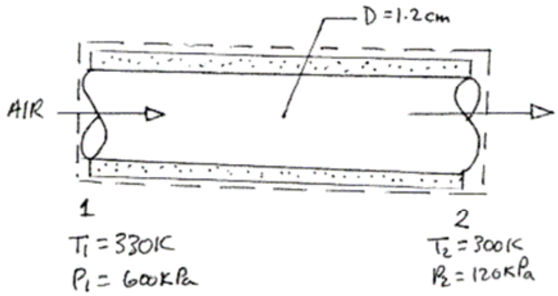
$$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{P_2 A_2 V_2}{RT_2}$$

$$= \frac{(120 \times 10^3 \frac{N}{m^2})\left(\frac{\pi}{4}\left(\frac{1.2}{100} m\right)^2\right)(251.73 m/s)}{\left(\frac{8314}{28.97} \frac{N \cdot m}{kg \cdot K}\right)(300K)} = 0.04 \frac{kg}{s} \qquad (b)$$

PROBLEM 4.27

Air at 600 kPa, 330 K enters a well-insulated, horizontal pipe having a diameter of 1.2 cm and exits at 120 kPa, 300 K. Applying the ideal gas model for air, determine at steady state (a) the inlet and exit velocities, each in m/s, and (b) the mass flow rate, in kg/s.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer can be ignored. Also, $\Delta pe = 0$ (horizontal); $\dot{W}_{cv} = 0$.
3. The air can be modeled as an ideal gas

ANALYSIS: (a) Mass rate balance, $\dot{m}_1 = \dot{m}_2$. That is

$$\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2} \Rightarrow \quad V_1 = \frac{v_1}{v_2} V_2 = \frac{(RT_1/P_1)}{(RT_2/P_2)} V_2$$

$$V_1 = \left(\frac{P_2}{P_1}\right)\left(\frac{T_1}{T_2}\right) V_2 = \left(\frac{120kPa}{600kPa}\right)\left(\frac{330K}{300K}\right) V_2$$

$$V_1 = 0.22\, V_2 \qquad (1)$$

Energy rate balance,

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g\cancel{(z_1 - z_2)}\right]$$

$$\Rightarrow \quad 0 = (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} \Rightarrow \quad 0 = (h_1 - h_2) + \frac{(0.22 V_2)^2 - V_2^2}{2}$$

Or, with data from Table A-22 for h_1 and h_2,

$$\frac{0.9516 V_2^2}{2} = (330.34 - 300.19)\frac{kJ}{kg}$$

$$\Rightarrow \quad V_2 = \sqrt{\frac{2(330.34 - 300.19)\ kJ/kg}{0.9516}\left|\frac{1\ kg\cdot m/s^2}{1\ N}\right|\left|\frac{10^3 N\cdot m}{1\ kJ}\right|}$$

$$= 251.73\ m/s \qquad (a)$$

And with Eq. (1) $\quad V_1 = 0.22\, V_2 = 55.38\ m/s$

(b) Then, with

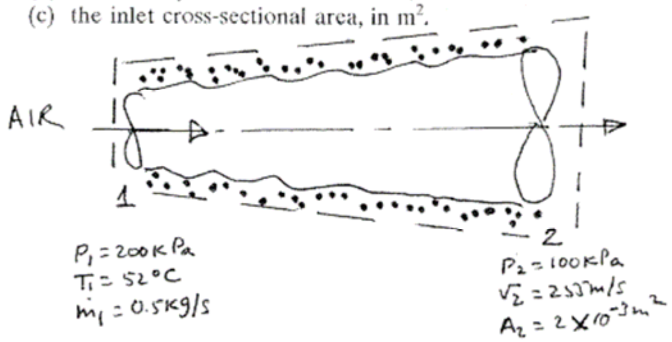
$$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{P_2 A_2 V_2}{RT_2}$$

$$= \frac{\left(120 \times 10^3 \frac{N}{m^2}\right)\left(\frac{\pi}{4}\left(\frac{1.2}{100} m\right)^2\right)\left(251.73 m/s\right)}{\left(\frac{8314}{28.97}\ \frac{N\cdot m}{kg\cdot K}\right)(300K)} = 0.04\ \frac{kg}{s} \qquad (b)$$

PROBLEM 4.28

At steady state, air at 200 kPa, 52°C and a mass flow rate of 0.5 kg/s enters an insulated duct having differing inlet and exit cross-sectional areas. At the duct exit, the pressure of the air is 100 kPa, the velocity is 255 m/s, and the cross-sectional area is 2×10^{-3} m². Assuming the ideal gas model, determine

(a) the temperature of the air at the exit, in °C.
(b) the velocity of the air at the inlet, in m/s.
(c) the inlet cross-sectional area, in m².



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and stray heat transfer can be ignored. Also, $\Delta pe = 0$.
3. The air is modeled as an ideal gas.

ANALYSIS: (a) Mass rate balance, $\dot{m}_2 = \dot{m}_1$. Thus, with $\dot{m}_2 = A_2 V_2 / v_2$ and $v_2 = RT_2/P_2$, we get

$$T_2 = \frac{A_2 V_2 P_2}{\dot{m} R} = \frac{(2\times10^{-3}\,m^2)(255\,m/s)(10^5\,N/m^2)}{\left(0.5\frac{kg}{s}\right)\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)} = 355.4\,K\ (82°C) \quad \longleftarrow (a)$$

(b) Energy rate balance,

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad V_1 = \sqrt{V_2^2 + 2(h_2 - h_1)}$$

with specific enthalpy data from Table A-22

$$V_1 = \sqrt{\left(255\frac{m}{s}\right)^2 + 2\,(355.94 - 325.31)\frac{kJ}{kg}\left|\frac{10^3\,N\cdot m}{1\,kJ}\right|\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right|}$$

$$= 355\,m/s \quad \longleftarrow (b)$$

(c) Since $\dot{m}_1 = A_1 V_1 / v_1 = \dfrac{P_1 A_1 V_1}{RT_1}$,

$$A_1 = \frac{\dot{m}_1 R T_1}{P_1 V_1} = \frac{(0.5\,kg/s)\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(325\,K)}{\left(200\times10^3\frac{N}{m^2}\right)(355\,m/s)}$$

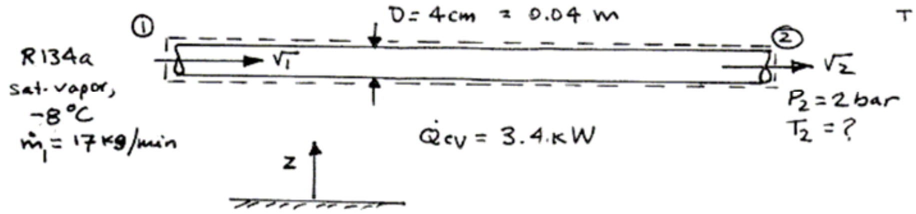
$$= 6.57\times10^{-4}\,m^2 \quad \longleftarrow (c)$$

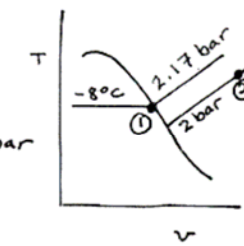
PROBLEM 4.29

KNOWN: Refrigerant 134a flows through a horizontal pipe at steady state, for which operating data are provided.

FIND: Determine the exit temperature and velocity, and the inlet velocity.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{W}_{cv} = 0$ and there is no change in potential energy from inlet to exit.

ANALYSIS: At steady state, the mass rate balance reduces to $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. The inlet velocity is found as follows:

$$\dot{m} = \frac{A_1 V_1}{v_1} \Rightarrow V_1 = \frac{\dot{m} v_1}{A_1} = \frac{4 \dot{m} v_1}{\pi d^2}$$

with v_1 from Table A-10

$$V_1 = \frac{4(17 \text{ kg/min}) \left|\frac{1 \text{ min}}{60 \text{ s}}\right| (0.0919 \text{ m}^3/\text{kg})}{\pi (0.04 \text{ m})^2} = 20.72 \text{ m/s} \longleftarrow V_1$$

Similarly, V_2 is found from $\dot{m}_1 = \dot{m}_2$ as follows:

$$\dot{m}_1 = \dot{m}_2 \Rightarrow \frac{\cancel{A_1} V_1}{v_1} = \frac{\cancel{A_2} V_2}{v_2} \Rightarrow V_2 = \frac{v_2}{v_1} V_1 \qquad (*)$$

In this expression, v_1 and V_1 are known. However, $v_2 = v(T_2, P_2)$ is unknown.

Another relation is obtained using the energy rate balance

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(\cancel{z_1 - z_2}^{\,0})\right]$$

$$= \dot{Q}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\right]$$

Inserting known values, including h_1 from Table A-10

$$0 = (3.4 \text{ kW})\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right| + \left(17 \frac{\text{kg}}{\text{min}}\right)\left|\frac{1 \text{ min}}{60 \text{ s}}\right|\left[(242.54 - h_2)\frac{\text{kJ}}{\text{kg}} + \left(\frac{(20.72 \text{ m/s})^2 - V_2^2}{2}\right)\left|\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|\right] \qquad (**)$$

where $h_2 = h(T_2, P_2)$.

Equations (*) and (**) can be solved simultaneously by referring to Table A-12 for v_2 and h_2 as functions of T_2 and P_2. The process is iterative. To avoid iteration, IT can be used effectively, as follows:

Continued on next slide

Problem 4-29 continued

IT Code

```
// Data
T1 = -8 // °C
x1 = 1
p2 = 2 // bar
d = 4 // cm
mdot = 17 // kg/min
Qdot = 3.4 // kW

// Determine mdot
p1 = Psat_T("R134A", T1)
v1 = vsat_Px("R134A", p1, x1)
h1 = hsat_Px("R134A", p1, x1)
mdot = ((pi * (d / 100)^2 / 4) * V1 / v1) * 60

// Find exit state
0 = Qdot + (mdot / 60) * ((h1 - h2) + (V1^2 - V2^2) / (2 * 1000))
V2 / v2 = V1 / v1
h2 = h_PT("R134A", p2, T2)
v2 = v_PT("R134A", p2, T2)
```

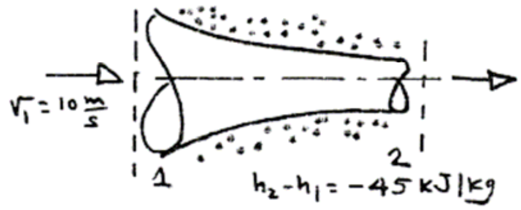
IT Results

1 $T_2 = 4.976°C \approx 5°C$ ← T_2
$V_2 = 24.08$ m/s ← V_2
$V_1 = 20.71$ m/s

PROBLEM 4.30

Steam enters a well-insulated, horizontal nozzle operating at steady state with a velocity of 10 m/s. If the specific enthalpy decreases by 45 kJ/kg from inlet to exit, determine the velocity at the exit, in m/s.

Schematic & Given Data:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and $\Delta pe = 0$.

ANALYSIS: Reducing Eq. 4.20a

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g\cancel{(z_1 - z_2)}\right] \Rightarrow V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

$$\therefore \quad V_2 = \sqrt{\left(10\frac{m}{s}\right)^2 + 2\left(45\frac{kJ}{kg}\right)\left|\frac{10^3\,N\cdot m}{1\,kJ}\right|\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right|} = 300\,m/s$$

PROBLEM 4.31

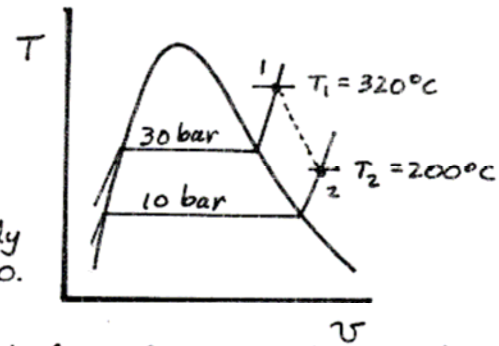
KNOWN: Steam flows through a nozzle with known conditions at the inlet and exit. The mass flow rate is given.

FIND: Determine (a) the exit velocity, (b) the inlet and exit flow areas.

SCHEMATIC & GIVEN DATA:

$P_1 = 30$ bar
$T_1 = 320°C$
$V_1 = 100$ m/s

$P_2 = 10$ bar
$T_2 = 200°C$
$\dot{m} = 2$ kg/s



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible and $\dot{W}_{cv} = 0$. (3) Potential energy effects are negligible.

ANALYSIS: (a) The velocity of steam at the exit is found from the steady-state energy balance

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancel{(z_1 - z_2)}^{\,0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for V_2

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2}$$

From Table A-4, $h_1 = 3043.4$ kJ/kg and $h_2 = 2827.9$ kJ/kg. Thus

$$V_2 = \sqrt{2(3043.4 - 2827.9)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right| + (100^2)\ \text{m}^2/\text{s}^2}$$

$$= 664.1\ \text{m/s} \longleftarrow \qquad V_2$$

(b) To find the inlet and exit flow areas, use $\dot{m} = (AV)/v$. Solving

$$A_1 = \frac{\dot{m}v_1}{V_1} \quad \text{and} \quad A_2 = \frac{\dot{m}v_2}{V_2}$$

From Table A-4, $v_1 = 0.0850$ m³/kg and $v_2 = 0.2060$ m³/kg. Thus

$$A_1 = \frac{(2\ \text{kg/s})(0.0850\ \text{m}^3/\text{kg})}{(100\ \text{m/s})}\left|\frac{10^4\ \text{cm}^2}{1\ \text{m}^2}\right| = 17\ \text{cm}^2 \longleftarrow \qquad A_1$$

and

$$A_2 = \frac{(2)(0.2060)}{(664.1)}\left|\frac{10^4}{1}\right| = 6.2\ \text{cm}^2 \longleftarrow \qquad A_2$$

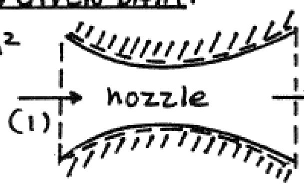
PROBLEM 4.32

KNOWN: Steam flows through a well-insulated nozzle with known conditions at the inlet and exit.

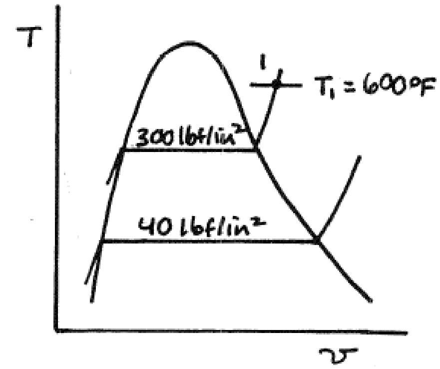
FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:

$p_1 = 300$ lbf/in^2
$T_1 = 600$°F
$V_1 = 100$ ft/s



$p_2 = 40$ lbf/in^2
$V_2 = 1800$ ft/s



ENGR. MODEL: (1) The control volume is at steady state. (2) There is no heat transfer, and $\dot{W}_{cv} = 0$. (3) Potential energy effects are negligible.

ANALYSIS: The pressure is known at the exit. The state is fixed by determining h_2 using the steady-state energy balance

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancelto{0}{(z_1 - z_2)}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for h_2

$$h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

From Table A-4E, $h_1 = 1314.5$ Btu/lb. Thus

$$h_2 = (1314.5 \text{ Btu/lb}) + \left(\frac{100^2 - 1800^2}{2}\right)\frac{\text{ft}^2}{\text{s}^2}\left|\frac{1 \text{ lbf}}{32.2 \text{ lb}\cdot\text{ft/s}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|$$

$$= 1250.03 \text{ Btu/lb}$$

Interpolating in Table A-4E at $p_2 = 40$ lbf/in^2, $h_2 = 1250.03$ Btu/lb gives

$$T_2 = 428 \text{ °F} \longleftarrow T_2$$

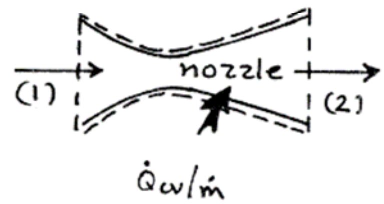
PROBLEM 4.33

KNOWN: Air flows through a nozzle with known conditions at the inlet and exit. Heat transfer occurs from the air to the surroundings.

FIND: Determine the heat transfer per lb of air flowing.

SCHEMATIC & GIVEN DATA:

$T_1 = 760°R$
$V_1 \approx 0$



$T_2 = 520°R$
$V_2 = 1500\ ft/s$

ENGR. MODEL: (1) The control volume is at steady state. (2) For the control volume, $\dot{W}_{cv} = 0$. (3) The air behaves as an ideal gas. (4) The inlet kinetic energy and potential energy effects are negligible.

ANALYSIS: To determine the heat transfer, begin with the steady-state energy balance

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{\cancel{V_1^2}^{\,0} - V_2^2}{2}\right) + g(\cancel{z_1 - z_2}^{\,0})\right]$$

Solving for $\dot{Q}_{cv}/\dot{m}$

$$\frac{\dot{Q}_{cv}}{\dot{m}} = h_2 - h_1 + \frac{V_2^2}{2}$$

From Table A-22E, $h_1 = 182.08$ Btu/lb and $h_2 = 124.27$ Btu/lb. Thus

$$\frac{\dot{Q}_{cv}}{\dot{m}} = (124.27 - 182.08)\frac{Btu}{lb} + \frac{\left(1500\frac{ft}{s}\right)^2}{2}\left|\frac{1\ lbf}{32.2\ ft\ lb/s^2}\right|\left|\frac{1\ Btu}{778\ ft\ lbf}\right|$$

$$\frac{\dot{Q}_{cv}}{\dot{m}} = -12.9\ Btu/lb \longleftarrow \dot{Q}/\dot{m}$$

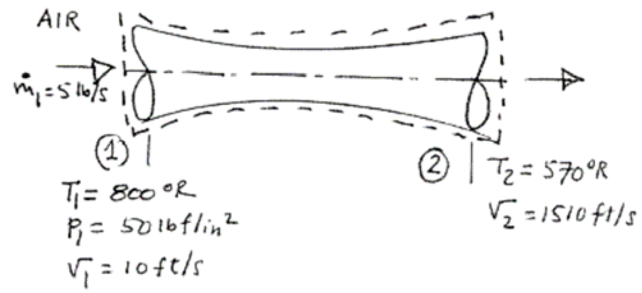
The negative sign indicates that the heat transfer is <u>from</u> the nozzle to the surroundings.

PROBLEM 4.34

KNOWN: Steady-state operating data are provided for a nozzle.

FIND: Evaluate the area at the inlet and $\dot{Q}_{cv}/\dot{m}$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. As shown by the sketch, a control volume encloses the nozzle.
2. The control volume is at steady state.
3. There is no change in potential energy from inlet to exit (horizontal).
4. Air is modeled as an ideal gas.

ANALYSIS: (a) Mass rate balance: $\dot{m}_2 = \dot{m}_1$, where $\dot{m}_1 = 5$ lb/s. Using

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} \Rightarrow A_1 = \frac{\dot{m}_1 \overbrace{v_1}^{=RT/p_1}}{V_1} = \frac{\dot{m} R T_1}{p_1 V_1} = \frac{(5\,\text{lb/s})\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(800°\text{R})}{\left(50 \times 144\,\frac{\text{lbf}}{\text{ft}^2}\right)\left(10\,\frac{\text{ft}}{\text{s}}\right)} = 2.96\ \text{ft}^2 \leftarrow A_1$$

(b) Energy rate balance: $0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right]$

$$\Rightarrow \frac{\dot{Q}_{cv}}{\dot{m}} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2}$$

Data from Table A-22E, $h_1 = 191.81$ Btu/lb, $h_2 = 136.26$ Btu/lb. Then

$$\frac{\dot{Q}_{cv}}{\dot{m}} = (136.26 - 191.81)\frac{\text{Btu}}{\text{lb}} + \left[\frac{(1510\ \text{ft/s})^2 - (10\ \text{ft/s})^2}{2}\right]\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

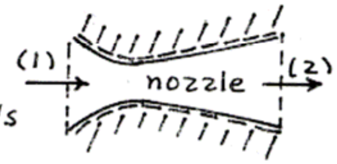
$$= -10.04\ \text{Btu/lb} \qquad \leftarrow \dot{Q}_{cv}/\dot{m}$$

PROBLEM 4.35

KNOWN: Helium flows through a well-insulated nozzle with known conditions at the inlet and exit. The mass flow rate is given.

FIND: Determine the exit area.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible and $\dot{W}_{cv} = 0$. (3) The helium behaves as an ideal gas. (4) Potential energy effects are negligible.

ANALYSIS: From the mass rate balance, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. With $\dot{m} = (AV)/v$

$$A_2 = \frac{\dot{m}\, v_2}{V_2}$$

Introducing the ideal gas equation of state

$$A_2 = \frac{\dot{m}\, R T_2}{V_2\, p_2}$$

The exit velocity is found using the steady-state energy balance

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancel{(z_1 - z_2)}^{0}\right]$$

From Table A-21, the specific heat c_p of helium is

$$c_p = (5/2)R = (5/2)\frac{(1.986\ \text{Btu/lbmol}\cdot°R)}{(4.003\ \text{lb/lbmol})} = 1.24\ \text{Btu/lb}\cdot°R$$

Since c_p is a constant $\Delta h = c_p \Delta T$, and

$$V_2 = \sqrt{2 c_p (T_1 - T_2) + V_1^2}$$

$$= \sqrt{2\left(1.24 \frac{\text{Btu}}{\text{lb}\cdot°R}\right)(600 - 460)°R \left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right|\left|\frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}}\right| + (175^2)\frac{\text{ft}^2}{\text{s}^2}}$$

$$= 2954\ \text{ft/s}$$

Finally

$$A_2 = \frac{(1\ \text{lb/s})\left(\frac{1545}{4.003}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(460°R)}{(2954\ \text{ft/s})(50\ \text{lbf/in}^2)}\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right|$$

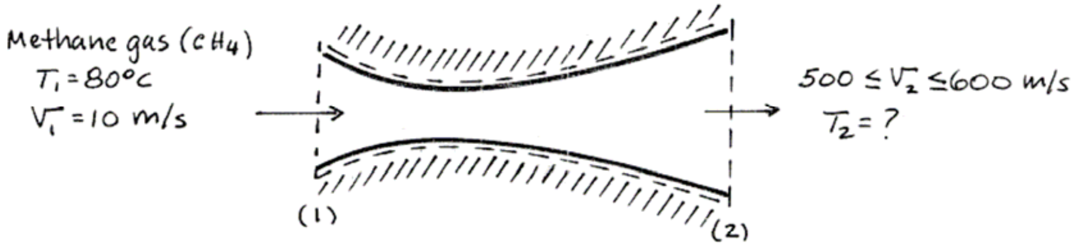
$$= 8.35 \times 10^{-3}\ \text{ft}^2 \longleftarrow A_2$$

PROBLEM 4.36

KNOWN: Methane gas flows through a nozzle with known inlet conditions and a specified range of exit velocities.

FIND: Plot exit temperature versus exit velocity.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (3) Potential energy effects can be ignored. (4) The methane behaves as an ideal gas.

ANALYSIS: Since $h = h(T)$ for the ideal gas, the exit temperature is determined by evaluating h_2 using steady state mass and energy balances:

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancel{(z_1 - z_2)}^{0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Thus, with $h = h(T)$

$$0 = h(T_1) - h(T_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\left|\frac{1\ N}{1\ kg \cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N \cdot m}\right| \qquad (*)$$

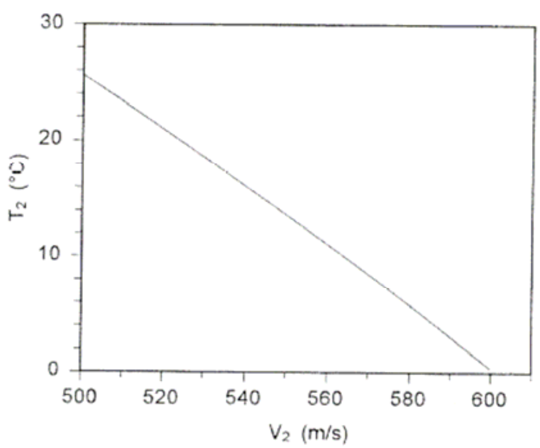
for h in kJ/kg and V in m/s.

Using IT, the functions $h(T)$ for methane (CH_4) as an ideal gas are accessed readily, and data for T_2 is obtained using the following code. The results are shown on the accompanying plot.

IT Code

```
T1 = 80  // °C
V1 = 10  // m/s
V2 = 550  // m/s

0 = (h1 - h2) + ((V1^2 - V2^2) / 2) * (1 / 10^3)  // kJ/kg
h1 = h_T("CH4", T1)
h2 = h_T("CH4", T2)

// Using the Explore button, sweep V2 from
// 500 to 600 in steps of 0.1.
```

1 Result for V2 = 550: T2 = 13.63°C



From (*), $h(T_2) = h(T_1) + \left(\frac{V_1^2 - V_2^2}{2}\right)$. Thus, as V_2 increases, $h(T_2)$ decreases. Therefore, T_2 decreases as expected.

---

1. A sample calculation using the c_p function from Table A-21 to evaluate the enthalpy change in (*) confirms this result. With IT, this integration is not necessary.

PROBLEM 4.37

KNOWN: Steady-state operating data are provided for a jet engine.

FIND: Determine the velocity at the diffuser exit.

SCHEMATIC & GIVEN DATA:

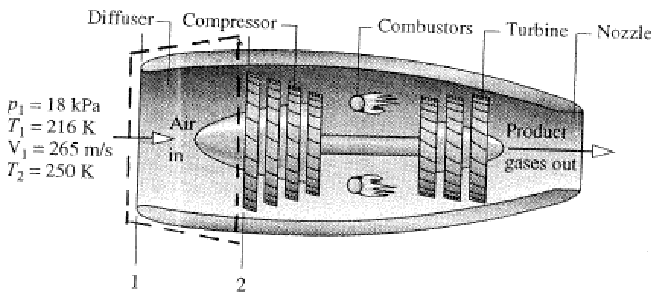


Fig. P4.37

ENGR. MODEL

1. As shown in the sketch, a control volume encloses the diffuser.
2. The control volume is at steady state.
3. For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$ and potential energy effects can be ignored.
4. The air is modeled as an ideal gas.

ANALYSIS: For the control volume, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_a$. An energy rate balance reads

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_a\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + \cancel{g(z_1 - z_2)}\right]$$

Accordingly,

$$V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

$$= \sqrt{\left(265 \frac{m}{s}\right)^2 + 2\,(\underbrace{215.97 - 250.05}_{\text{Table A-22}})\frac{kJ}{kg}\left|\frac{10^3 N\cdot m}{1\,kJ}\right|\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right|}$$

$$= 45 \text{ m/s}$$

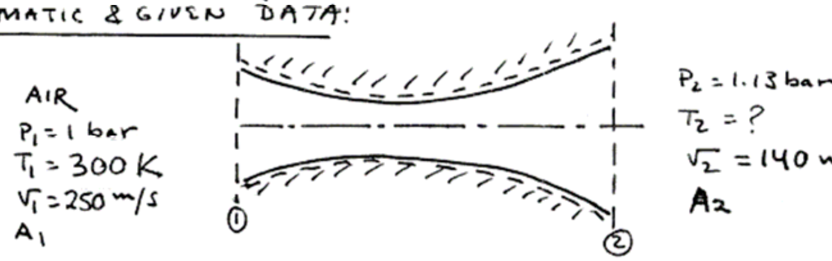
V_2

PROBLEM 4.38

KNOWN: Data is provided for a diffuser at steady state, through which air is flowing.

FIND: Determine the ratio of the exit flow area to the inlet flow area, and the exit temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and potential energy effects can be ignored. 3. Air is modeled as an ideal gas.

ANALYSIS: The mass rate balance reads $\dot{m}_2 = \dot{m}_1$, or

$$\frac{A_2 V_2}{v_2} = \frac{A_1 V_1}{v_1} \Rightarrow \frac{A_2 V_2}{(RT_2/P_2)} = \frac{A_1 V_1}{(RT_1/P_1)} \Rightarrow \frac{A_2}{A_1} = \left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right) \qquad (1)$$

The exit temperature, T_2, can be obtained using an energy rate balance:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g\,\cancel{(z_1 - z_2)}\right]$$

$$h_2 = h_1 + \frac{V_1^2}{2} - \frac{V_2^2}{2}$$

Using data from Table A-22

$$h_2 = \left(300.19\,\frac{kJ}{kg}\right) + \left(\frac{250^2 - 140^2}{2}\right)\frac{m^2}{s^2}\left|\frac{1\,N}{1\,kg\,m/s^2}\right|\left|\frac{1\,kJ}{1000\,Nm}\right|$$

$$= 321.64\ kJ/kg$$

Then, interpolating in Table A-22 for $h_2 = 321.64$ kJ/kg

$$T_2 = 321.3\ K \longleftarrow \quad T_2$$

Returning to Eq. (1)

$$\frac{A_2}{A_1} = \left(\frac{1\,bar}{1.13\,bar}\right)\left(\frac{321.3\,K}{300\,K}\right)\left(\frac{250\,m/s}{140\,m/s}\right)$$

$$= 1.692 \quad A_2/A_1$$

PROBLEM 4.39

KNOWN: Refrigerant 134a passes through an insulated diffuser. Data are known at the inlet and exit.

FIND: Determine the exit temperature.

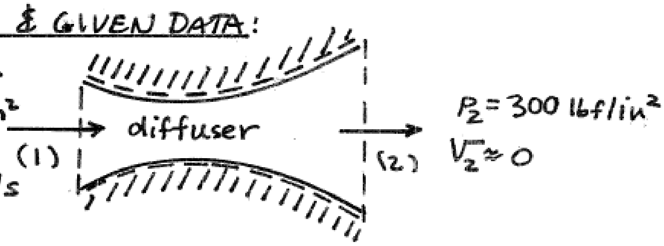
SCHEMATIC & GIVEN DATA:

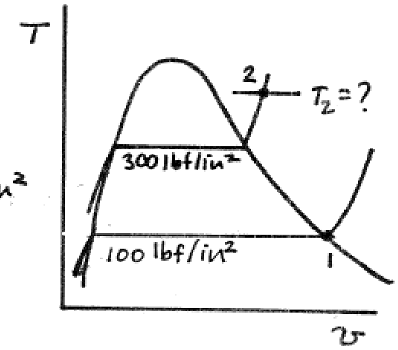
R-134a

$p_1 = 100\ \text{lb/in}^2$

Sat. vapor

$V_1 = 1200\ \text{ft/s}$





ENGR. MODEL: (1) The control volume is at steady state. (2) The heat transfer is negligible and $\dot{W}_{cv} = 0$. (3) Potential energy effects and kinetic energy at exit can be neglected.

ANALYSIS: The exit pressure is given. The state is fixed by determining h_2 using a steady-state energy balance

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - \cancel{V_2^2}^{0}}{2}\right) + g(\cancel{z_1 - z_2}^{0})\right]$$

where $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Solving for h_2

$$h_2 = h_1 + \frac{V_1^2}{2}$$

From Table A-11E, at $p_1 = 100\ \frac{\text{lbf}}{\text{in}^2}$, $h_1 = h_g = 112.46$ Btu/lb. Thus

$$h_2 = 112.46\ \text{Btu/lb} + \frac{1}{2}(1200^2\ \text{ft}^2/\text{s}^2)\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= 141.20\ \text{Btu/lb}$$

Interpolating in Table A-12E at $p_2 = 300\ \text{lbf/in}^2$, $h_2 = 141.20$ Btu/lb gives

$$T_2 \approx 222.9\ ^\circ\text{F} \longleftarrow T_2$$

PROBLEM 4.40

KNOWN: Carbon dioxide flows through a well-insulated diffuser with known conditions at the inlet and exit.

FIND: Determine the exit temperature and pressure and the mass flow rate.

SCHEMATIC & GIVEN DATA:

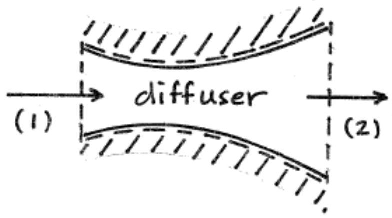
CO_2

$p_1 = 20$ lbf/in^2

$T_1 = 500$°R

$V_1 = 800$ ft/s

$A_1 = 1.4$ in^2



$A_2/A_1 = 30$

$V_2 = 20$ ft/s

ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible and $\dot{W}_{cv} = 0$. (3) Potential energy effects are negligible. (4) The carbon dioxide behaves as an ideal gas.

ANALYSIS: Since $h = h(T)$ for an ideal gas, the exit temperature can be found by evaluating h_2. Beginning with the steady-state energy balance

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancel{(z_1 - z_2)}^{0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for h_2, and noting that $\bar{h} = h/M$

$$h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right) \Rightarrow \bar{h}_2 = \bar{h}_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)M$$

where M is the molecular weight. From Table A-23E, $\bar{h}_1 = 3706.2$ Btu/lbmol and

$$\bar{h}_2 = 3706.2\ \frac{\text{Btu}}{\text{lbmol}} + \left(\frac{800^2 - 20^2}{2}\right)\frac{\text{ft}^2}{\text{s}^2}\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{44.01\ \text{lb}}{1\ \text{lbmol}}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= 4268\ \text{Btu/lbmol}$$

Interpolating in Table A-23E; $T_2 \approx 563.6$°R ← T_2

To get p_2, begin with $\dot{m}_1 = \dot{m}_2$. Thus

$$\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

with $pv = RT$

$$\frac{p_1}{RT_1}(A_1 V_1) = \frac{p_2}{RT_2}(A_2 V_2)$$

or

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)\left(\frac{A_1}{A_2}\right)\left(\frac{V_1}{V_2}\right) = \left(20\ \frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{563.6}{500}\right)\left(\frac{1}{30}\right)\left(\frac{800}{20}\right)$$

$$= 30.06\ \text{lbf/in}^2$$ ← p_2

Finally, the mass flow rate is

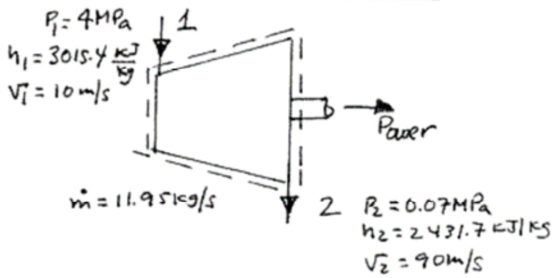
$$\dot{m} = \frac{(A_1 V_1)p_1}{RT_1} = \frac{(1.4\ \text{in}^2)(800\ \text{ft/s})(20\ \text{lbf/in}^2)}{\left(\frac{1545}{44.01}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(500°\text{R})}$$

$$= 1.276\ \text{lb/s}$$ ← $\dot{m}$

PROBLEM 4.41

. . . Steam enters a well-insulated turbine operating at steady state at 4 MPa with a specific enthalpy of 3015.4 kJ/kg and a velocity of 10 m/s. The steam expands to the turbine exit where the pressure is 0.07 MPa, specific enthalpy is 2431.7 kJ/kg, and the velocity is 90 m/s. The mass flow rate is 11.95 kg/s. Neglecting potential energy effects, determine the power developed by the turbine, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$ and $\Delta pe \approx 0$.

ANALYSIS: Reducing Eq. 4.20a

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right)$$

$$\Rightarrow \dot{W}_{cv} = \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right]$$

$$= \left(11.95\ \frac{kg}{s}\right)\left[\left(3015.4\ \frac{kJ}{kg} - 2431.7\ \frac{kJ}{kg}\right) + \left[\frac{\left(10\ \frac{m}{s}\right)^2 - \left(90\ \frac{m}{s}\right)^2}{2}\right]\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|\right]$$

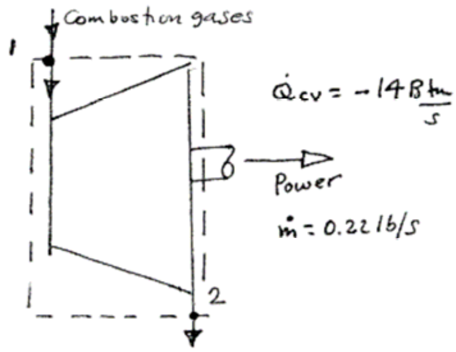
$$= 11.95\ \frac{kg}{s}\left[583.7\ \frac{kJ}{kg} - 4\ \frac{kJ}{kg}\right]\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 6927\ kW$$

PROBLEM 4.42

Hot combustion gases, modeled as air behaving as an ideal gas, enter a turbine at 145 lbf/in.^{2}, 2700°R with a mass flow rate of 0.22 lb/s and exit at 29 lbf/in.^{2} and 1620°R. If heat transfer from the turbine to its surroundings occurs at a rate of 14 Btu/s, determine the power output of the turbine, in hp.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The control volume shown in the figure is at steady state.
2. Kinetic and potential energy effects are ignored.
3. The combustion gases are modeled as air as an ideal gas.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)} \right]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}(h_1 - h_2)$$

with enthalpy data from Table A-22E,

$$\dot{W}_{cv} = -14\,\frac{Btu}{s} + 0.22\,\frac{lb}{s}\left[703.35 - 401.10\right]\frac{Btu}{lb}$$

$$= 52.495\,\frac{Btu}{s}\left|\frac{3600\,s}{1\,h}\right|\left|\frac{1\,hp}{2545\,Btu/h}\right|$$

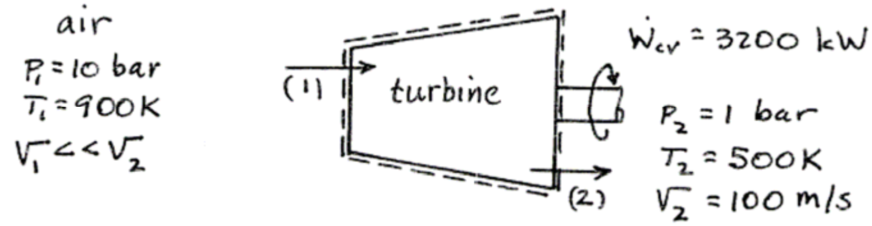
$$= 74.26\,hp$$

PROBLEM 4.43

KNOWN: Air expands through a turbine with known conditions at the inlet and exit. The power developed is known.

FIND: Determine the mass flow rate and the exit area.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) Potential energy effects and kinetic energy at the inlet can
① be neglected. (4) The air behaves as an ideal gas.

ANALYSIS: Begin with a steady-state energy balance

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{\cancelto{0}{V_1^2} - V_2^2}{2}\right) + g\,\cancelto{0}{(z_1 - z_2)}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{cv}}{(h_1 - h_2) - \frac{V_2^2}{2}}$$

From Table A-22; $h_1 = 932.93$ kJ/kg and $h_2 = 503.02$ kJ/kg. Thus

$$\dot{m} = \frac{(3200\ \text{kW})\ 1\ \text{kJ/s}/1\ \text{kW}}{(932.93 - 503.02)\ \text{kJ/kg} - \left(\frac{100^2\ \text{m}^2/\text{s}^2}{2}\right)\left|\frac{1\ \text{N}}{\text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|}$$

$$= 7.53\ \text{kg/s} \longleftarrow \dot{m}$$

The exit area is

$$A_2 = \frac{v_2 \dot{m}}{V_2} = \frac{R T_2 \dot{m}}{p_2 V_2}$$

$$= \frac{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(500\text{K})(7.53\ \text{kg/s})}{(1\ \text{bar})(100\ \text{m/s})}\left|\frac{1\ \text{bar}}{10^5\ \text{N/m}^2}\right|\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right|$$

$$= 0.108\ \text{m}^2 \longleftarrow A_2$$

---

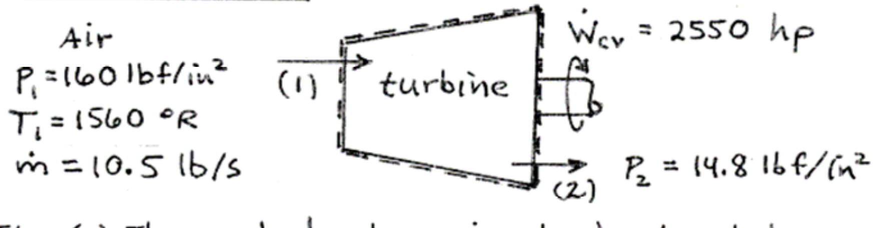
1. The applicability of the ideal gas model can be checked by reference to the compressibility chart.

PROBLEM 4.44

KNOWN: Air expands through a turbine with known conditions at the inlet and exit. The inlet mass flow rate and the power developed are given.

FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) Kinetic and potential energy effects are negligible.
① (4) The air behaves as an ideal gas.

ANALYSIS: Since $h = h(T)$ for an ideal gas, the exit temperature can be found by evaluating h_2. Beginning with the steady-state energy balance

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \cancel{g(z_1 - z_2)}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$, and with assumption (3) we get

$$0 = -\dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

or

$$h_2 = -\dot{W}_{cv}/\dot{m} + h_1 \qquad (1)$$

Using data from Table A-22E for h_1 and inserting values into (1)

$$h_2 = -\frac{(2550\ \text{hp})}{(10.5\ \text{lb/s})}\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|\left|\frac{2545\ \text{Btu/h}}{1\ \text{hp}}\right| + 385.08\ \frac{\text{Btu}}{\text{lb}}$$

$$= 213.39\ \text{Btu/lb}$$

Interpolating in Table A-22E

$$T_2 = 888.3\ °R \longleftarrow T_2$$

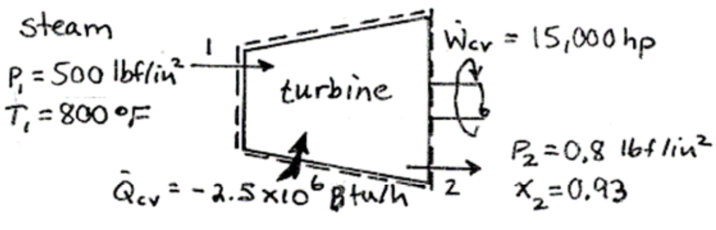
1. The applicability of the ideal gas model can be checked by reference to the compressibility chart.

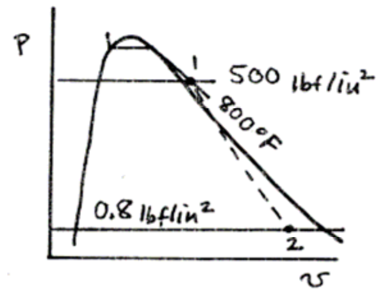
PROBLEM 4.45

KNOWN: A steam turbine operates at steady state with known inlet and exit conditions. The power developed and the heat transfer rate between the turbine and its surroundings are specified.

FIND: Determine the volumetric flow rate of steam at the inlet.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: (1) The control volume is at steady state. (2) Kinetic and potential energy changes from inlet to exit can be neglected.

ANALYSIS: To calculate the volume flow rate, begin with mass and energy rate balances for the one-inlet, one-exit control volume at steady state

$$0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{0} + g\cancel{(z_1 - z_2)}^{0}\right]$$

Solving for the mass flow rate

$$\dot{m} = \frac{\dot{Q}_{cv} - \dot{W}_{cv}}{(h_2 - h_1)}$$

From Table A-4E, at $p_1 = 500$ lbf/in², $T_1 = 800$°F; $h_1 = 1412.1$ Btu/lb. From Table A-3E, at $p_2 = 0.8$ lbf/in²

$$h_2 = h_{f2} + x_2 h_{fg2}$$
$$= (62.41) + (0.93)(1040.2) = 1029.80 \text{ Btu/lb}$$

Thus

$$\dot{m} = \frac{(-2.5\times10^6 \text{ Btu/h}) - (15{,}000 \text{ hp})\left|\frac{2545 \text{ Btu/h}}{1 \text{ hp}}\right|}{(1029.80 - 1412.1) \text{ Btu/lb}}$$

$$= 10.64\times10^4 \text{ lb/h}$$

Then, from Table A-4E at p_1 and T_1; $v_1 = 1.441$ ft³/lb.

The volumetric flow rate at the inlet is then

$$(AV)_1 = \dot{m}\, v_1 = \left(10.64\times10^4 \frac{\text{lb}}{\text{h}}\right)\left(1.441 \frac{\text{ft}^3}{\text{lb}}\right)$$

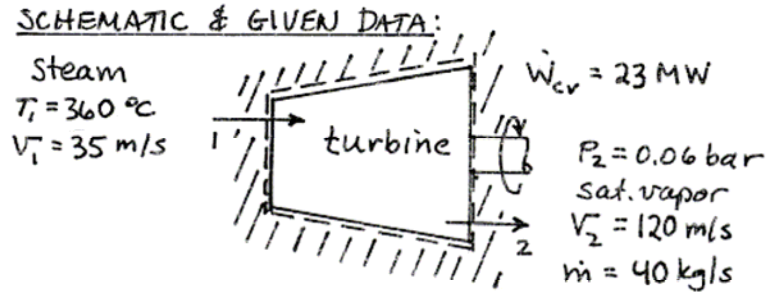
$$= 1.53\times10^5 \text{ ft}^3/\text{h} \longleftarrow (AV)_1$$

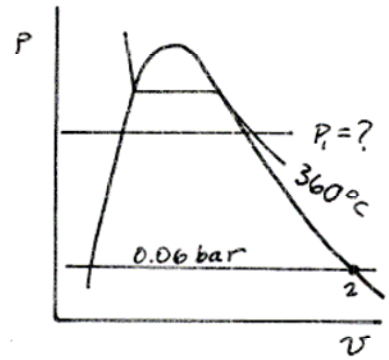
PROBLEM 4.46

KNOWN: A well-insulated steam turbine operates at steady-state with known inlet and exit conditions. The power output and mass flow rate are specified.

FIND: Determine the inlet pressure.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer can be neglected. (3) The potential energy change from inlet to exit can be neglected.

ANALYSIS: The temperature is known at the inlet. To fix the inlet state, h_1 is determined using mass and energy balances. For the control volume

$$\cancelto{0}{\frac{dm_{cv}}{dt}} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

and

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancelto{0}{(z_1 - z_2)}\right]$$

Solving for h_1

$$h_1 = \frac{\dot{W}_{cv}}{\dot{m}} + h_2 + \left(\frac{V_2^2 - V_1^2}{2}\right)$$

From Table A-3 for saturated vapor at $p_2 = 0.06$ bar; $h_2 = 2567.4$ kJ/kg. Thus

$$h_1 = \frac{(23\ MW)}{(40\ kg/s)}\left|\frac{10^3\ kW}{1\ MW}\right| + \left(2567.4\ \frac{kJ}{kg}\right) + \left(\frac{120^2 - 35^2}{2}\right)\frac{m^2}{s^2}\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 3149.0\ kJ/kg$$

Interpolating in Table A-4, with $T_1 = 360\ °C$ and $h_1 = 3149.0$ kJ/kg gives

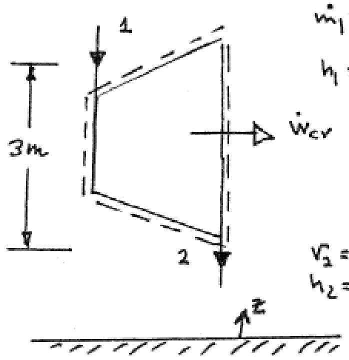
$$p_1 \approx 25.0\ bar \longleftarrow p_1$$

PROBLEM 4.47

KNOWN: Steady-state data are provided for a steam turbine.

FIND: Determine the power developed by the turbine.

SCHEMATIC & GIVEN DATA



$\dot{m}_1 = 10\frac{kg}{min}$, $V_1 = 30 m/s$

$h_1 = 3100 kJ/kg$

$\frac{\dot{Q}_{cv}}{\dot{m}} = -1.1\frac{kJ}{kg}$

$V_2 = 45 m/s$

$h_2 = 2300 kJ/kg$

ENGR. MODEL:

1. As shown in the sketch, a control volume encloses the turbine.
2. The control volume is at steady state.
3. $g = 9.81 m/s^2$.

ANALYSIS: Mass rate balance at steady state, $\dot{m}_2 = \dot{m}_1$. Energy rate balance at steady state,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right]$$

$$\Rightarrow \quad \frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right]$$

where

✓ $\frac{\dot{Q}_{cv}}{\dot{m}} = -1.1\frac{kJ}{kg}$

✓ $(h_1 - h_2) = (3100 - 2300)\frac{kJ}{kg} = 800\frac{kJ}{kg}$

✓ $\frac{V_1^2 - V_2^2}{2} = \frac{[(30 m/s)^2 - (45 m/s)^2]}{2}\left|\frac{1 N}{1 kg\cdot m/s^2}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right| = -0.56\frac{kJ}{kg}$

✓ $g(z_1 - z_2) = \left(9.81\frac{m}{s^2}\right)(3 m)\left|\frac{1 N}{1 kg\cdot m/s^2}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right| = 0.03\frac{kJ}{kg}$

Collecting results,

$$\frac{\dot{W}_{cv}}{\dot{m}} = \left[-1.1 + 800 - 0.56 + 0.03\right]\frac{kJ}{kg}$$

$$= 798.37\frac{kJ}{kg}$$

$$\Rightarrow \quad \dot{W}_{cv} = \left(10\frac{kg}{min}\right)\left(798.3\frac{kJ}{kg}\right)\left|\frac{1 min}{60 s}\right|\left|\frac{1 kW}{1 kJ/s}\right|$$

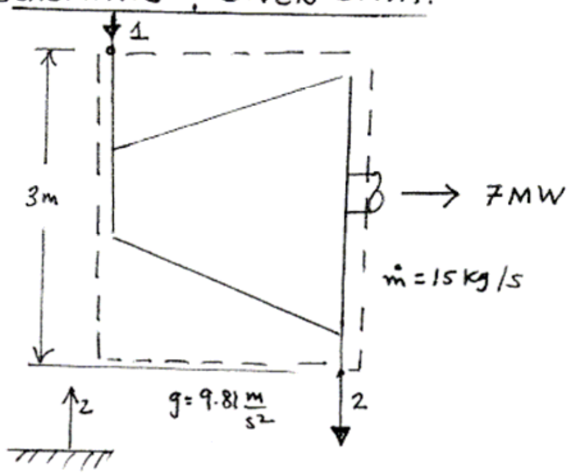
$$= 133.1 \text{ kW}$$

$\dot{W}_{cv}$

PROBLEM 4.48

... Steam enters a turbine operating at steady state at 2 MPa, 360°C with a velocity of 100 m/s. Saturated vapor exits at 0.1 MPa and a velocity of 50 m/s. The elevation of the inlet is 3 m higher than at the exit. The mass flow rate of the steam is 15 kg/s, and the power developed is 7 MW. Let $g = 9.81$ m/s^2. Determine (a) the area at the inlet, in m^2, and (b) the rate of heat transfer between the turbine and its surroundings, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. $g = 9.81\ m/s^2$

ANALYSIS:

(a) $\dot{m} = \frac{A_1 V_1}{v_1} \Rightarrow A_1 = \frac{\dot{m} v_1}{V_1} = \frac{(15 kg/s)(0.1411 m^3/kg)}{(100 m/s)} = 0.021 m^2$ (a) — Table A-4

(b) Considering Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right]$$

$$\Rightarrow \quad \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right] \qquad (1)$$

where, with data from Tables A-3 and A-4,

$$h_2 - h_1 = (2675.5 - 3159.3)\frac{kJ}{kg} = -483.8\frac{kJ}{kg}$$

$$\frac{V_2^2 - V_1^2}{2} = \left[\frac{(50\ m/s)^2 - (100\ m/s)^2}{2}\right]\left|\frac{1 N}{1 kg \cdot m/s^2}\right|\left|\frac{1 kJ}{10^3 N \cdot m}\right| = -3.75\frac{kJ}{kg}$$

$$g(z_2 - z_1) = \left(9.8\frac{m}{s^2}\right)(-3 m)\left|\frac{1 N}{1 kg \cdot m/s^2}\right|\left|\frac{1 kJ}{10^3 N \cdot m}\right| = -0.03\frac{kJ}{kg}$$

Inserting values in Eq. (1)

$$\dot{Q}_{cv} = 7000\ kW + 15\frac{kg}{s}\left[-483.8 - 3.75 - 0.03\right]\frac{kJ}{kg}\left|\frac{1 kW}{1 kJ/s}\right|$$

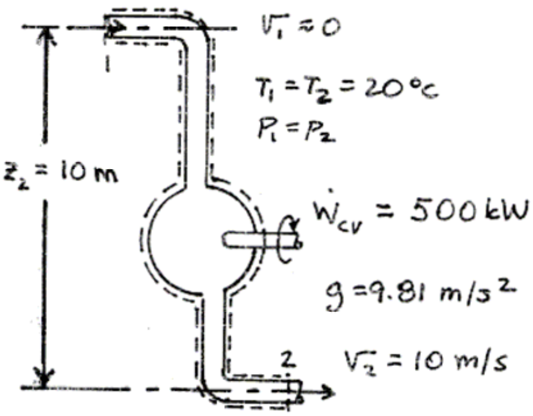
$$= -313.7\ kW \qquad (b)$$

PROBLEM 4.49

KNOWN: Water flows through a hydraulic turbine with known conditions at the inlet and exit. The power output is specified.

FIND: Determine the mass flow rate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) Changes in temperature and pressure from inlet to exit are negligible. (4) Kinetic energy can be neglected at the inlet. (5) The acceleration of gravity is constant; $g = 9.81$ m/s².

ANALYSIS: To find the mass flow rate, begin with steady state mass and energy rate balances

$$0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

and

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[\cancelto{0}{(h_1 - h_2)} + \left(\frac{\cancelto{0}{V_1^2} - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

where the enthalpy term is cancelled because of assumption (3). Solving

$$\dot{m} = \frac{\dot{W}_{cv}}{-\frac{V_2^2}{2} + g(z_1 - z_2)}$$

Inserting values

$$\dot{m} = \frac{(500\ \text{kW})\left|\frac{1\ \text{kJ/s}}{1\ \text{kW}}\right|}{\left[\left(-\frac{10^2}{2}\right)\frac{\text{m}^2}{\text{s}^2} + (9.81\ \frac{\text{m}}{\text{s}^2})(10\ \text{m})\right]\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|}$$

$$= 10{,}400\ \text{kg/s} \longleftarrow \dot{m}$$

PROBLEM 4.50

KNOWN: Steady-state operating data are provided for a two-stage turbine with a reheater.

FIND: Determine the steam mass flow rate, the total power developed, and rate of heat transfer for the steam flowing through the reheater.

SCHEMATIC & GIVEN DATA:

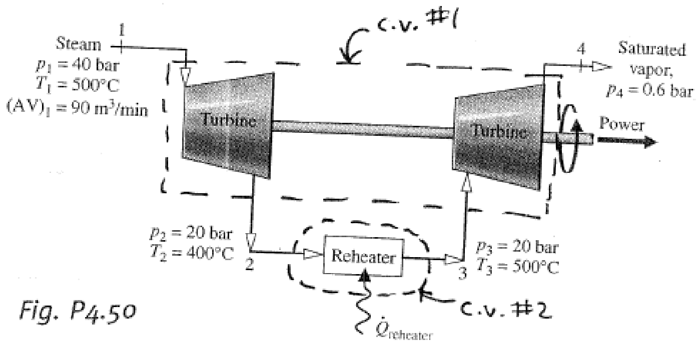


Fig. P4.50

ENGR. MODEL:

1. As shown in the sketch, two control volumes are under consideration.
2. Each control volume is at steady state.
3. Kinetic and potential energy effects can be ignored.
4. For control volume #1, stray heat transfer can be ignored.

ANALYSIS: (a) The mass flow rate at inlet 1 is found from $\dot{m}_1 = \frac{(AV)_1}{v_1}$.
From Table A-4 at 40 bar, 500°C, $v_1 = 0.08643\ m^3/kg$. Then,

$$\dot{m}_1 = \frac{90\ m^3/min}{0.08643\ m^3/kg}\left|\frac{60\ min}{1\ h}\right| = 6.248 \times 10^4 \frac{kg}{h} \qquad \longleftarrow \dot{m}$$

(b) An energy rate balance for control volume #1 reads

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}[h_1 - h_2] + \dot{m}[h_3 - h_4]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{m}\left[(h_1 - h_2) + (h_3 - h_4)\right)$$

With data from Tables A-3 and A-4, $h_1 = 3445.3\ kJ/kg$, $h_2 = 3247.6\ kJ/kg$, $h_3 = 3467.6\ kJ/kg$, $h_4 = 2653.5\ kJ/kg$.

$$\Rightarrow \quad \dot{W}_{cv} = \dot{m}\left[(3445.3 - 3247.6) + (3467.6 - 2653.5)\right]\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 6.248 \times 10^4 \frac{kg}{h}\left|\frac{1\ h}{3600\ s}\right| = 17.36 \frac{kg}{s}$$

$$= 17{,}565\ kW \qquad \longleftarrow \dot{W}_{cv}$$

(c) An energy rate balance for control volume #2 reads

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}(h_2 - h_3) \Rightarrow \dot{Q}_{cv} = \dot{m}(h_3 - h_2)$$

$$\Rightarrow \quad \dot{Q}_{cv} = 17.36 \frac{kg}{s}(3467.6 - 3247.6)\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

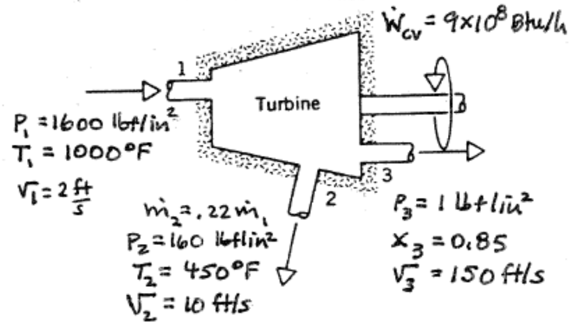
$$= 3{,}819\ kW \qquad \longleftarrow \dot{Q}_{cv}$$

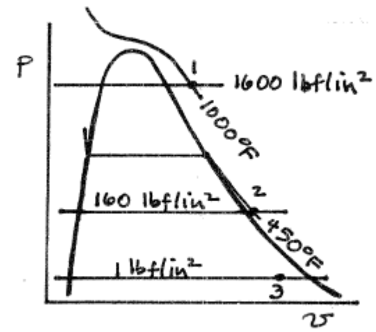
PROBLEM 4.51

KNOWN: Steam passes through an extraction turbine operating at steady state with know inlet and exit conditions. The power output is specified.

FIND: Determine (a) the inlet mass flow rate, (b) the diameter of the extraction duct.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: 1. A control volume enclosing the turbine is at steady state. 2. For the control volume, heat transfer and potential energy effects are negligible.

ANALYSIS: (a) To find $\dot{m}_1$, apply a mass rate balance: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$. Then, since $\dot{m}_2/\dot{m}_1 = 0.22$, we have $\dot{m}_3/\dot{m}_1 = 0.78$. Next, apply an energy rate balance:

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} + \dot{m}_1\left[h_1 + \frac{V_1^2}{2}\right] - \dot{m}_2\left[h_2 + \frac{V_2^2}{2}\right] - \dot{m}_3\left(h_3 + \frac{V_3^2}{2}\right]$$

where the potential energy terms are omitted by assumption 2. Solving

$$\dot{m}_1 = \frac{\dot{W}_{cv}}{\left[h_1 + \frac{V_1^2}{2}\right] - \frac{\dot{m}_2}{\dot{m}_1}\left[h_2 + \frac{V_2^2}{2}\right] - \frac{\dot{m}_3}{\dot{m}_1}\left[h_3 + \frac{V_3^2}{2}\right]}$$

From Table A-4E, $h_1 = 1487$ Btu/lb, $h_2 = 1246.1$ Btu/lb. With Table A-3E data $h_3 = h_{f3} + x_3(h_{g3} - h_{f3}) = 69.74 + 0.85(1036) = 950.3$ Btu/lb. Thus

$$\dot{m}_1 = \frac{(9.\times10^8 \text{ Btu/h})}{\left[1487\frac{\text{Btu}}{\text{lb}} + \frac{(2\text{ ft/s})^2}{2}\left|\frac{1\text{ lbf}}{32.2\text{ lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|\right] - 0.22\left[1246.1 + \frac{(10)^2}{(2)|32.2||778|}\right] - 0.78\left[950.3 + \frac{(150)^2}{(2)|32.2||778|}\right]}$$

$= 1.91\times10^6$ lb/h ← $\dot{m}_1$

(b) Using $v_2 = 3.228$ from Table A-4E

$$A_2 = \frac{\dot{m}_2 v_2}{V_2} = \frac{(.22)(1.91\times10^6\text{ lb/h})(3.228\text{ ft}^3/\text{lb})}{(10\text{ ft/s})}\left|\frac{1\text{ h}}{3600\text{ s}}\right| = 37.68\text{ ft}^2$$

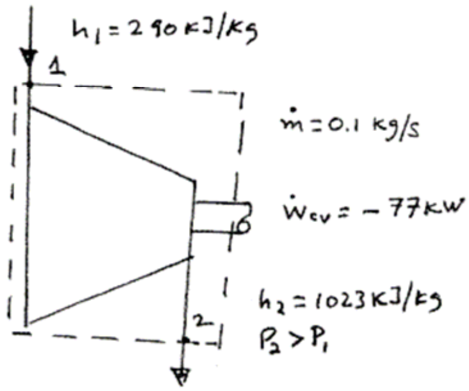
Since $A_2 = \pi d_2/4$

$$d_2 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{(4)(37.68\text{ ft}^2)}{\pi}} = 6.93\text{ ft}$$ ← d_2

PROBLEM 4.52

Air enters a compressor operating at steady state at 1 atm with a specific enthalpy of 290 kJ/kg and exits at a higher pressure with a specific enthalpy of 1023 kJ/kg. The mass flow rate is 0.1 kg/s. If the compressor power input is 77 kW, determine the rate of heat transfer between the compressor and its surroundings, in kW. Neglect kinetic and potential energy effects.

SCHEMATIC & GIVEN DATA



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Kinetic and potential energy effects can be neglected.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}\left[h_2 - h_1\right]$$

$$= -77\ kW + 0.1\ kg/s\left[1023 - 290\right]\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= -3.7\ kW$$

PROBLEM 4.53

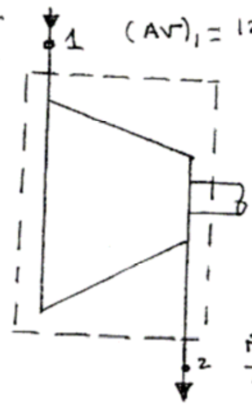
Air enters a compressor operating at steady state at 1.05 bar, 300 K, with a volumetric flow rate of 12 m^3/min and exits at 12 bar, 400 K. Heat transfer occurs at a rate of 2 kW from the compressor to its surroundings. Assuming the ideal gas model for air and neglecting kinetic and potential energy effects, determine the power input, in kW.

SCHEMATIC & GIVEN DATA:

$p_1 = 1.05$ bar
$T_1 = 300$K

$(AV)_1 = 12$ ft^3/min

$\dot{Q}_{cv} = -2$kW



$p_2 = 12$ bar
$T_2 = 400$K

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. The air is modeled as an ideal gas.
3. Kinetic and potential energy effects are neglected.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}[h_1 - h_2] \qquad (1)$$

where

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 p_1}{RT_1} = \frac{(12 \text{ ft}^3/\text{min})(1.05 \times 10^5 \text{ N/m}^2)}{\left(\frac{8314}{28.97} \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300\text{K})} \left|\frac{1 \text{ min}}{60 \text{ s}}\right|$$

$$= 0.2439 \frac{\text{kg}}{\text{s}}$$

Then, with enthalpy data from Table A-22, Eq. (1) gives

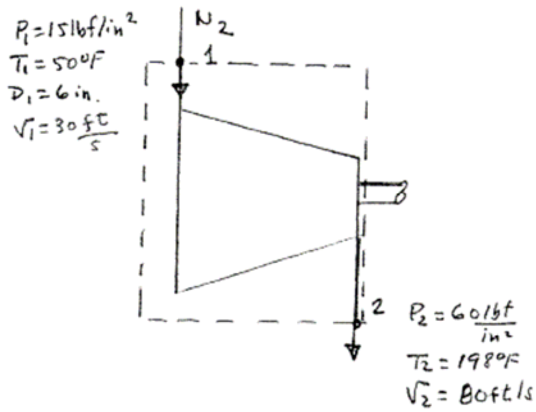
$$\dot{W}_{cv} = -2\text{kW} + \left(0.2439 \frac{\text{kg}}{\text{s}}\right)[300.19 - 400.98] \frac{\text{kJ}}{\text{kg}} \left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= -26.6 \text{ kW}$$

PROBLEM 4.54

Nitrogen is compressed in an axial-flow compressor operating at steady state from a pressure of 15 lbf/in.^{2} and a temperature of 50°F to a pressure 60 lbf/in.^{2}. The gas enters the compressor through a 6-in.-diameter duct with a velocity of 30 ft/s and exits at 198°F with a velocity of 80 ft/s. Using the ideal gas model, and neglecting stray heat transfer and potential energy effects, determine the compressor power input, in hp.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and potential energy effects can be ignored.
3. The nitrogen is modeled as an ideal gas.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right]$$

where

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{(\pi D_1^2/4)(V_1)}{RT_1/P_1} = \frac{P_1(\pi D_1^2/4)V_1}{RT_1} = \frac{\left(15 \times 144\,\frac{lbf}{ft^2}\right)\left(\frac{\pi}{4}(0.5\,ft)^2\right)\left(30\,\frac{ft}{s}\right)}{\left(\frac{1545}{28.01}\,\frac{ft\cdot lbf}{lb\cdot °R}\right)(510°R)}$$

$$= 0.45\,lb/s$$

Then with specific enthalpies on a molar basis from Table A-23E,

$$\dot{W}_{cv} = \left(0.45\,\frac{lb}{s}\right)\left[\frac{(3541.8 - 4571.9)}{28.01}\,\frac{Btu}{lb} + \left(\frac{(30)^2 - (80)^2}{2}\left(\frac{ft^2}{s^2}\right)\left|\frac{1\,lbf}{32.2\,lb\cdot ft/s^2}\right|\left|\frac{1\,Btu}{778\,ft\cdot lbf}\right|\right)\right]$$

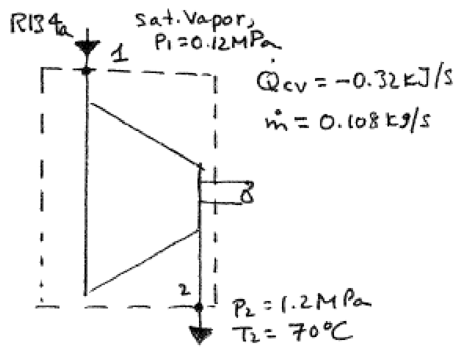
$$= \left(0.45\,\frac{lb}{s}\right)\left[-36.78 - 0.11\right]\frac{Btu}{lb}\left|\frac{3600\,s}{h}\right|\left|\frac{1\,hp}{2545\,Btu/h}\right|$$

$$= -23.48\,hp$$

PROBLEM 4.55

Refrigerant 134a enters a compressor operating at steady state as saturated vapor at 0.12 MPa and exits at 1.2 MPa and 70°C at a mass flow rate of 0.108 kg/s. As the refrigerant passes through the compressor, heat transfer to the surroundings occurs at a rate of 0.32 kJ/s. Determine at steady state the power input to the compressor, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, kinetic and potential energy effects can be ignored.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}\left[h_1 - h_2\right]$$

With data from Tables A-11 and A-12,

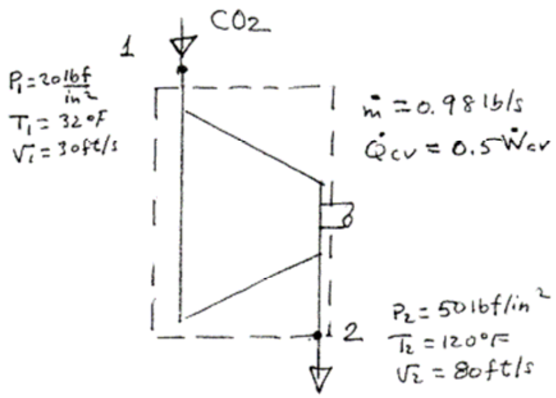
$$\dot{W}_{cv} = -0.32\frac{kJ}{s} + 0.108\frac{kg}{s}\left[233.86 - 298.96\right]\frac{kJ}{kg}$$

$$= -7.35\frac{kJ}{s}\left|\frac{1\ kW}{1\ kJ/s}\right| = -7.35\ kW$$

PROBLEM 4.56

Carbon dioxide gas is compressed at steady state from a pressure of 20 lbf/in.^{2} and a temperature of 32°F to a pressure of 50 lbf/in.^{2} and a temperature of 120°F. The gas enters the compressor with a velocity of 30 ft/s and exits with a velocity of 80 ft/s. The mass flow rate is 0.98 lb/s. The magnitude of the heat transfer rate from the compressor to its surroundings is 5% of the compressor power input. Using the ideal gas model with $c_p = 0.21$ Btu/lb · °R and neglecting potential energy effects, determine the compressor power input, in horsepower.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, potential energy effects can be neglected.
3. The CO_2 is modeled as an ideal gas with constant $c_p = 0.21$ Btu/lb·°R.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g\cancel{(z_1 - z_2)}\right]$$

$\llcorner\ \dot{Q}_{cv} = 0.5\dot{W}_{cv}$

$$\Rightarrow \qquad \dot{W}_{cv} = \frac{\dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right]}{0.95} = \frac{\dot{m}\left[c_p(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2}\right]}{0.95}$$

Inserting values,

$$\dot{W}_{cv} = \frac{\left(0.98\frac{lb}{s}\right)\left[0.21\frac{Btu}{lb\cdot °R}(-88°R) + \left[\frac{(30)^2 - (80)^2}{2}\left(\frac{ft}{s}\right)^2\left|\frac{1\,lbf}{32.2\,lb\cdot ft/s^2}\right|\left|\frac{1\,Btu}{778\,ft\cdot lbf}\right|\right]\right]}{0.95}$$

$$= \frac{\left(0.98\frac{lb}{s}\right)\left[-18.48\frac{Btu}{lb} - 0.11\frac{Btu}{lb}\right]\left|\frac{3600\,s}{1\,h}\right|\left|\frac{1\,hp}{2545\,Btu/h}\right|}{0.95}$$

$$= -27.1 \text{ hp}$$

The power input is 27.1 hp. ◄

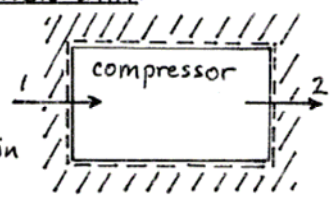
PROBLEM 4.57

KNOWN: A well-insulated air compressor operates with known inlet and exit states. The inlet volumetric flow rate is also known.

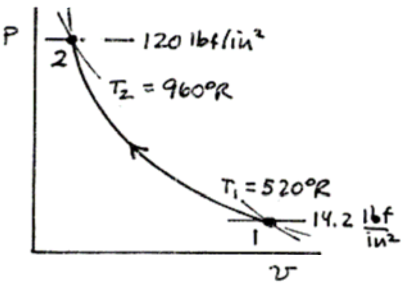
FIND: Determine the compressor power and the exit volumetric flow rate.

SCHEMATIC & GIVEN DATA:

air
$T_1 = 60°F$
$p_1 = 14.2\ lbf/in^2$
$(AV)_1 = 1200\ ft^3/min$



$T_2 = 500°F$
$p_2 = 120\ lbf/in^2$



ENGR. MODEL: (1) The control volume is at steady
① state. (2) Heat transfer is negligible. (3) The air behaves as an ideal gas. (4) Kinetic and potential energy changes from inlet to exit can be neglected.

ANALYSIS: To find the power, begin with steady state mass and energy balances

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{\,0} + g\cancel{(z_1 - z_2)}^{\,0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for $\dot{W}_{cv}$

$$\dot{W}_{cv} = \dot{m}\left[h_1 - h_2\right]$$

To evaluate $\dot{m}$, use Eq. 4.4b and the ideal gas equation of state

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1}$$

$$= \frac{(14.2\ lbf/in^2)(1200\ ft^3/min)}{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(520°R)}\left|\frac{60\ min}{1\ h}\right|\left|\frac{144\ in^2}{1\ ft^2}\right| = 5309\ lb/h$$

Thus, with $h_1 = 124.27$ Btu/lb and $h_2 = 231.06$ Btu/lb from Table A-22E

$$\dot{W}_{cv} = \left(5309\ \frac{lb}{h}\right)(124.27 - 231.06)\frac{Btu}{lb}\left|\frac{1\ hp}{2545\ Btu/h}\right|$$

② $= -222.8\ hp$ ← $\dot{W}_{cv}$

The exit volumetric flow rate is

$$(AV)_2 = \dot{m} v_2 = \dot{m}\left(\frac{RT_2}{p_2}\right)$$

$$= (5309\ lb/h)\left|\frac{1\ h}{60\ min}\right| \frac{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(960\ °R)}{(120\ lbf/in^2)}\left|\frac{1\ ft^2}{144\ in^2}\right|$$

$= 262\ ft^3/min$ ← $(AV)_2$

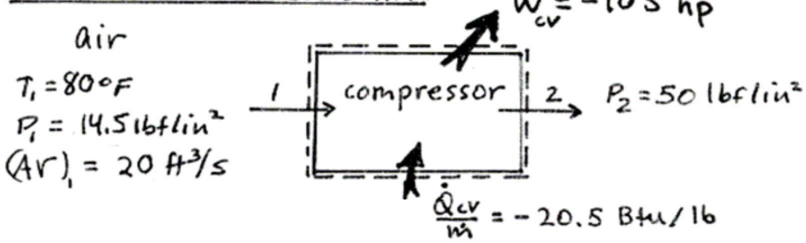
1. The applicability of the ideal gas model can be checked by reference to the generalized compressibility chart.
2. The negative sign for power denotes energy transfer into the control volume, as expected.

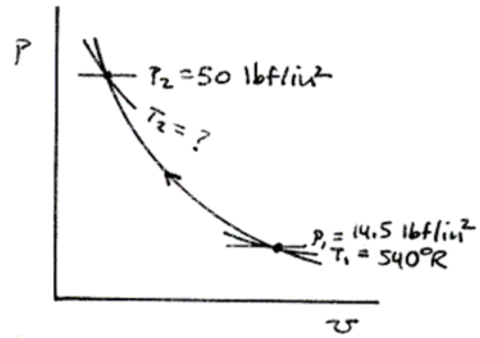
PROBLEM 4.58

KNOWN: An air compressor operates with known power, inlet conditions, and exit pressure. The inlet volumetric flow rate and the heat transfer rate per unit mass of air flow are also specified.

FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: (1) The control volume is at steady state. (2) Kinetic and
1 potential energy changes from inlet to exit are negligible. (3) The air behaves as an ideal gas.

ANALYSIS: To find the compressor power, begin with steady state mass and energy rate balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{0} + g\cancel{(z_1 - z_2)}^{0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for h_2

$$h_2 = h_1 + \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}}$$

To evaluate $\dot{m}$, use Eq. 4.4b and the ideal gas equation of state

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{RT_1}$$

$$= \frac{(14.5 \text{ lbf/in}^2)(20 \text{ ft}^3/\text{s})}{\left(\frac{1545}{28.97} \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(540°\text{R})} \left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right| = 1.44 \text{ lb/s}$$

Using data from Table A-22E

$$h_2 = \left(129.06 \frac{\text{Btu}}{\text{lb}}\right) + \left(-20.5 \frac{\text{Btu}}{\text{lb}}\right) - \left(\frac{-105 \text{ hp}}{1.44 \text{ lb/s}}\right)\left|\frac{1 \text{ h}}{3600 \text{ s}}\right|\left|\frac{2545 \text{ Btu/h}}{1 \text{ hp}}\right|$$

$$= 160.1 \text{ Btu/lb}$$

Interpolating in Table A-22E with $h_2 = 160.1$ Btu/lb

$$T_2 = 669°\text{R} = 209°\text{F} \longleftarrow T_2$$

---

1. The applicability of the ideal gas model can be checked using the compressibility chart.

PROBLEM 4.59

KNOWN: A Refrigerant 134a compressor has known conditions at the inlet and exit. The inlet volumetric flow rate and the compressor power per unit mass of refrigerant flowing are also specified.

FIND: Determine the heat transfer rate.

SCHEMATIC & GIVEN DATA:

Refrigerant 134a

$P_1 = 3.2$ bar

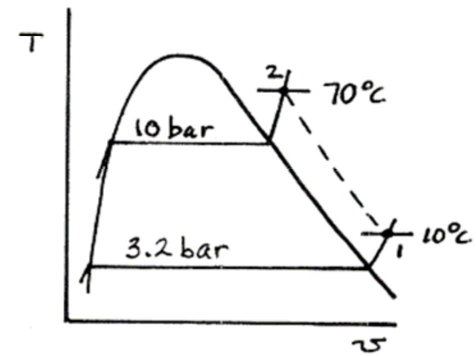
$T_1 = 10°C$

$(AV)_1 = 3.0$ m³/min

$P_2 = 10$ bar

$T_2 = 70$ °C

$\frac{\dot{W}_{cv}}{\dot{m}} = -55.2$ kJ/kg



ENGR. MODEL: (1) The control volume is at steady state. (2) Kinetic and potential energy changes from inlet to exit can be neglected.

ANALYSIS: To find the heat transfer rate, begin with steady state energy and mass rate balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{0} + g\cancel{(z_1 - z_2)}^{0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving

$$\dot{Q}_{cv} = \dot{m}\left[\left(\frac{\dot{W}_{cv}}{\dot{m}}\right) + (h_2 - h_1)\right]$$

To evaluate $\dot{m}$, use Eq. 4.4b and data for v_1 from Table A-12

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{3.0 \text{ m}^3/\text{min}}{0.06576 \text{ m}^3/\text{kg}} = 45.62 \text{ kg/min}$$

From Table A-12, $h_1 = 255.65$ kJ/kg, and $h_2 = 302.34$ kJ/kg

Thus

$$\dot{Q}_{cv} = \left(45.62 \frac{\text{kg}}{\text{min}}\right)\left[(-55.2) + (302.34 - 255.65)\right]\frac{\text{kJ}}{\text{kg}}\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right)$$

$$= -0.14 \text{ kW} \longleftarrow \dot{Q}_{cv}$$

The negative sign indicates that the heat transfer is <u>from</u> the system.

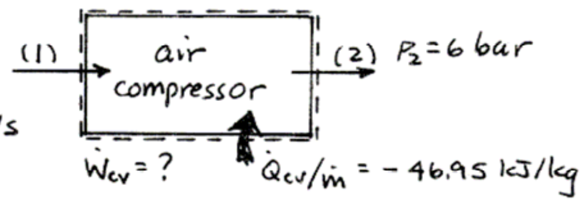
PROBLEM 4.60

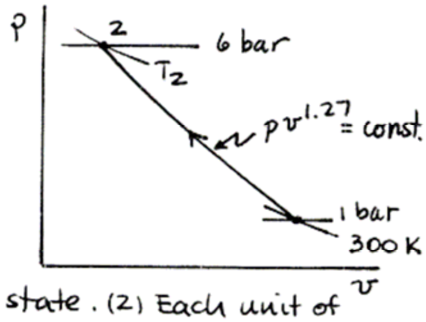
KNOWN: Air is compressed at steady state from a given initial state to a given final pressure. The mass flow is known, and each unit of mass undergoes a specified process in going from inlet to exit.

FIND: Determine the compressor power.

SCHEMATIC & GIVEN DATA:

$T_1 = 300\text{K}$
$P_1 = 1\text{ bar}$
$\dot{m} = 4\text{ kg/s}$





ENGR. MODEL: (1) The control volume is at steady state. (2) Each unit of
① mass undergoes a process described by $pv^{1.27} = $ constant. (3) The air can be modeled as an ideal gas. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: To determine the power, begin with mass and energy rate balances at steady state

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{0} + \cancel{g(z_1 - z_2)}^{0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$ and the indicated terms are deleted by assumption 4. Rearranging and solving for $\dot{W}_{cv}$

$$\dot{W}_{cv} = \dot{m}\left[(\dot{Q}_{cv}/\dot{m}) + (h_1 - h_2)\right] \qquad (*)$$

Now, specific enthalpy h_1 is read from Table A-22 at 300K: $h_1 = 300.19 \frac{\text{kJ}}{\text{kg}}$.
To get T_2, we use Eq. 3.56 with $n = 1.27$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} T_1 = \left(\frac{6}{1}\right)^{\frac{1.27-1}{1.27}} (300\text{K}) = 439.1\text{ K}$$

Interpolating in Table A-22; $h_2 = 440.7$ kJ/kg. Inserting values in (*)

$$\dot{W}_{cv} = \left(4\frac{\text{kg}}{\text{s}}\right)\left[\left(-46.95\frac{\text{kJ}}{\text{kg}}\right) + (300.19 - 440.7)\frac{\text{kJ}}{\text{kg}}\right]\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

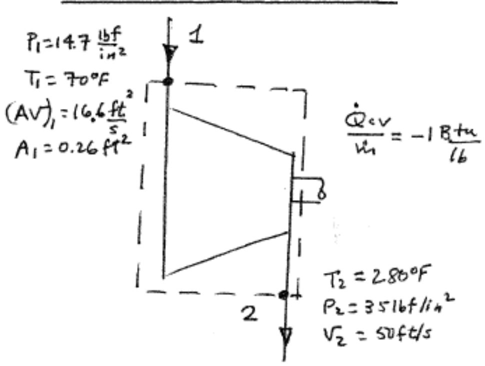
② $= -750$ kW ←——— $\dot{W}_{cv}$

1. The applicability of the ideal gas model can be checked by reference to the generalized compressibility chart.
2. The negative sign for power denotes energy transfer by work <u>into</u> the control volume, as expected.

PROBLEM 4.61

Air enters a compressor operating at steady state with a pressure of 14.7 lbf/in.^{2} and a temperature of 70°F. The volumetric flow rate at the inlet is 16.6 ft^3/s, and the flow area is 0.26 ft^2. At the exit, the pressure is 35 lbf/in.^{2}, the temperature is 280°F, and the velocity is 50 ft/s. Heat transfer from the compressor to its surroundings occurs at a rate of 1.0 Btu per lb of air flowing. Potential energy effects are negligible, and the ideal gas model can be assumed for the air. Determine (a) the velocity of the air at the inlet, in ft/s, (b) the mass flow rate, in lb/s, and (c) the compressor power, in Btu/s and hp.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, potential energy effects are negligible.
3. The air is modeled as an ideal gas.

ANALYSIS:

(a) Using given data at the inlet,

$$V_1 = \frac{(AV)_1}{A_1} = \frac{16.6\ ft^3/s}{0.26\ ft^2} = 63.85 \frac{ft}{s} \quad \longleftarrow \text{(a)}$$

(b)

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{RT_1} = \frac{\left(14.7 \times 144 \frac{lbf}{ft^2}\right)\left(16.6 \frac{ft^3}{s}\right)}{\left(\frac{1545}{28.97} \frac{ft \cdot lbf}{lb \cdot °R}\right)(530\ °R)} = 1.24 \frac{lb}{s} \quad \longleftarrow \text{(b)}$$

(c) Reducing Eq. 4.20a

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{m}\left[\frac{\dot{Q}_{cv}}{\dot{m}} + (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2}\right]$$

Inserting values from Table A-22E and other data,

$$\dot{W}_{cv} = 1.24 \frac{lb}{s}\left(-1 \frac{Btu}{lb} + (126.67 - 177.23) \frac{Btu}{lb} + \left[\frac{(63.85)^2 - (50)^2}{2}\right]\left(\frac{ft^2}{s^2}\right)\left|\frac{1\ lbf}{32.2\ lb \cdot ft/s^2}\right|\left|\frac{1\ Btu}{778\ ft \cdot lbf}\right|\right)$$

$$= 1.24 \frac{lb}{s}\left[-1 - 50.56 + 0.03\right] \frac{Btu}{lb}$$

$$= -63.9 \frac{Btu}{s} \quad \longleftarrow$$

or

$$\dot{W}_{cv} = -63.9 \frac{Btu}{s}\left|\frac{3600 s}{1 h}\right|\left|\frac{1 hp}{2545 Btu/h}\right|$$

$$= -90.4\ hp \quad \longleftarrow$$

PROBLEM 4.62

KNOWN: An ammonia compressor has known conditions at the inlet and exit. The compressor power and the heat transfer rate are also specified.

FIND: Determine the inlet volumetric flow rate using ammonia tables and ideal gas relationships. Discuss.

SCHEMATIC & GIVEN DATA:

Ammonia
$P_1 = 20\ \text{lbf/in}^2$
$T_1 = 0°F$

$P_2 = 250\ \text{lbf/in}^2$
$T_2 = 300°F$
$\dot{W}_{cv} = -10\ \text{hp}$
$\dot{Q}_{cv} = -5000\ \text{Btu/h}$

ENGR. MODEL: (1) The control volume is at steady state. (2) Kinetic and potential energy changes from inlet to exit are negligible. (3) For the second part, the ammonia behaves as an ideal gas.

ANALYSIS: To begin, determine the mass flow rate by using the steady-state mass and energy balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{0} + \cancel{g(z_1 - z_2)}^{0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving

$$\dot{m} = \frac{\dot{Q}_{cv} - \dot{W}_{cv}}{(h_2 - h_1)} \qquad (*)$$

—*—*—*—*—*—*—

From Table A-15E; $h_1 = 614.84$ Btu/lb and $h_2 = 760.39$ Btu/lb. Thus

$$\dot{m} = \frac{(-5000\ \text{Btu/h}) - (-10\ \text{hp})\left|\frac{2545\ \text{Btu/h}}{1\ \text{hp}}\right|}{(760.39 - 614.84)\ \text{Btu/lb}}\left|\frac{1\ \text{h}}{60\ \text{min}}\right| = 2.342\ \text{lb/min}$$

Now, using $v_1 = 14.078\ \text{ft}^3/\text{lb}$ from Table A-15E and Eq. 4-11b

$$(AV)_1 = \dot{m}\, v_1 = (2.342)(14.078) = 32.97\ \text{ft}^3/\text{min} \leftarrow \underset{\text{(Ammonia table)}}{(AV)_1}$$

——*—*—*—*—*—

To evaluate $h_2 - h_1$ for an ideal gas, integrate the specific heat function $c_p(T)$ for ammonia from Table A-21E

$$h_2 - h_1 = \frac{\bar{R}}{M}\int_{T_1}^{T_2}(\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4)\, dT$$

$$= \frac{\bar{R}}{M}\left[\alpha(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) + \frac{\gamma}{3}(T_2^3 - T_1^3) + \frac{\delta}{4}(T_2^4 - T_1^4) + \frac{\epsilon}{5}(T_2^5 - T_1^5)\right]$$

With $T_1 = 460°R$, $T_2 = 760°R$, and coefficient values from Table A-21E

$$h_2 - h_1 = \frac{(1.986\ \text{Btu/lbmol}\cdot°R)}{(17.04\ \text{lb/lbmol})}\left[1329\ °R\right]$$

$$= 154.9\ \text{Btu/lb}$$

Continued on next slide

Using this result with (*)

$$\dot{m} = \frac{(-5000) - (-10)(2545)}{(154.9)} \left| \frac{1}{60} \right| = 2.2 \text{ lb/min}$$

Incorporating the ideal gas equation of state into Eq. 4-11b

$$(AV)_1 = \dot{m} v_1 = \dot{m}\left(\frac{RT_1}{p_1}\right)$$

$$= \left(2.2 \frac{\text{lb}}{\text{min}}\right) \frac{\left(\frac{1545}{17.04} \cdot \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(460^\circ\text{R})}{(20 \text{ lbf/in}^2)} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right|$$

$$= 31.86 \text{ ft}^3/\text{min} \longleftarrow \frac{(AV)_1}{(\text{ideal gas})}$$

* * * *

Discussion: The % deviation in assuming ideal gas behavior is

$$\%\text{ deviation} = \left[\frac{(AV)_{tables} - (AV)_{ideal\,gas}}{(AV)_{tables}}\right] \times 100$$

$$= \left[\frac{32.97 - 31.86}{32.97}\right] \times 100 = 3.4\%$$

Thus, the ideal gas model is reasonably accurate. To explore this further, consider

$$\%\text{ deviation} = \left[\frac{\Delta h_{ideal\,gas} - \Delta h_{tables}}{\Delta h_{tables}}\right] \times 100$$

$$= \left[\frac{154.9 - 145.55}{145.55}\right] \times 100 = 6.4\%$$

The applicability of the ideal gas model can also be checked by determining the compressibility factor. For state 1

$$Z_1 = \frac{p_1 v_1}{RT_1} = \frac{(20)|144|(14.078)}{\left(\frac{1545}{17.04}\right)(460)} = 0.972$$

For state 2, $v_2 = 1.8191$ ft³/lb from Table A-15E. Thus

$$Z_2 = \frac{(250)|144|(1.8191)}{\left(\frac{1545}{17.04}\right)(760)} = 0.95$$

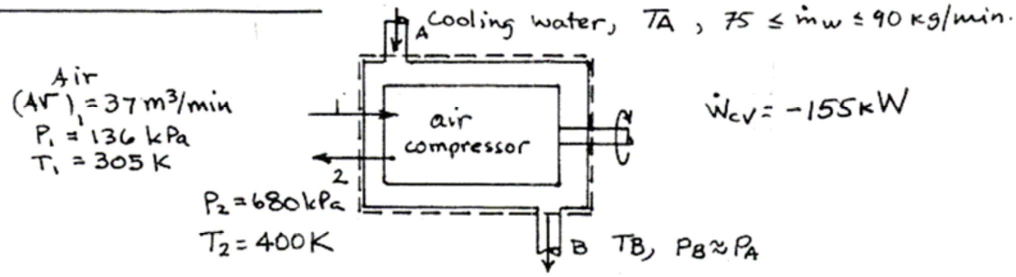
Both of these values are reasonably close to unity.

PROBLEM 4.63

KNOWN: Data are provided for a water-jacketed air compressor operating at steady state.

FIND: Determine the cooling water temperature increase for a specified cooling water mass flow rate. Plot the cooling water temperature increase versus cooling water mass flow rate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer from the outside of the cooling water jacket is negligible. (3) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas. (5) The cooling water is incompressible with $c_p = 4.179$ kJ/kg·K from Table A-19.

ANALYSIS: (a) The steady-state energy balance for the overall compressor is

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_1\left(h_1 + \underline{\frac{V_1^2}{2} + gz_1}\right) - \dot{m}_2\left(h_2 + \underline{\frac{V_2^2}{2} + gz_2}\right)$$
$$+ \dot{m}_A\left(h_A + \underline{\frac{V_A^2}{2} + gz_A}\right) - \dot{m}_B\left(h_B + \underline{\frac{V_B^2}{2} + gz_B}\right)$$

where the indicated terms drop out by assumptions (2) and (3). Since the water and air streams are separate

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_{air}$$
$$\dot{m}_A = \dot{m}_B \equiv \dot{m}_w$$

Thus

$$0 = -\dot{W}_{cv} + \dot{m}_{air}(h_1 - h_2) + \dot{m}_w(h_A - h_B) \qquad (1)$$

With assumption 5, the enthalpy change of the cooling water can be found using Eq. 3.20b:

$$h_B - h_A = c(T_B - T_A) + \cancel{v(P_B - P_A)} = c(T_B - T_A)$$

Accordingly, Eq. (1) gives

$$T_B - T_A = \frac{\left[-\dot{W}_{cv} + \dot{m}_{air}(h_1 - h_2)\right]}{c\,\dot{m}_w}$$

Using the data at location 1 to evaluate $\dot{m}_{air}$

$$\dot{m}_{air} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 P_1}{RT_1} = \frac{(37\ \text{m}^3/\text{min})(136\ \text{kPa})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(305\ \text{K})}\left|\frac{10^3\ \text{N/m}^2}{1\ \text{kPa}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$
$$= 57.49\ \text{kg/min}$$

Also, from Table A-22, $h_1 = 305.22$ kJ/kg, $h_2 = 400.48$ kJ/kg

Collecting results

$$T_B - T_A = \frac{\left[-(-155\ \text{kJ/s})(60\ \text{s/min}) + (57.49\ \text{kg/min})(305.22 - 400.48)\ \text{kJ/kg}\right]}{(4.179\ \text{kJ/kg}\cdot\text{K})\ \dot{m}_w}$$

Continued on next slide

Problem 4-63 continued

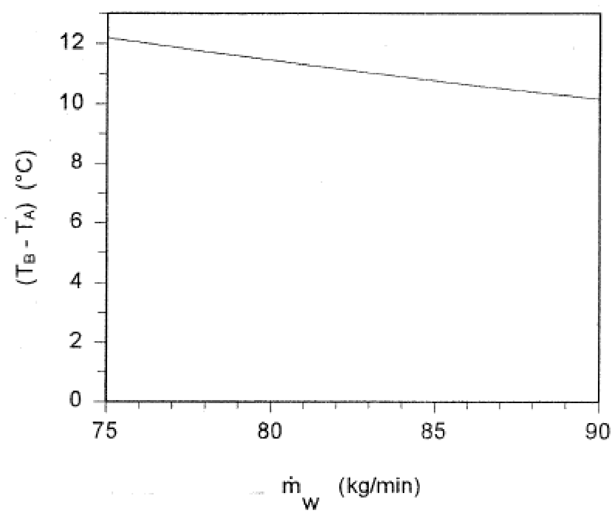
or

$$(T_B - T_A) = \frac{3823.5 \text{ kJ/min}}{(4.179 \text{ kJ/kg}\cdot\text{K})\,\dot{m}_w}$$

$$= \frac{914.93 \text{ kg}\cdot\text{K/min}}{\dot{m}_w} \qquad (*)$$

For the case of $\dot{m}_w = 82$ kg/min; $T_B - T_A = 11.2$ K ← ΔT_w (part a)

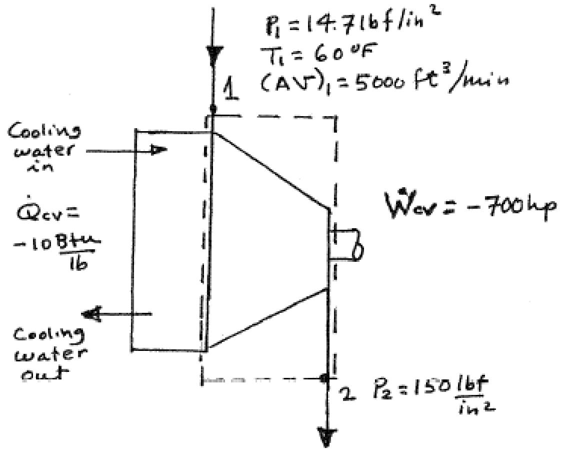
(b) Equation (*) can be plotted readily using a spreadsheet, plotting program, or IT. The IT plot follows:



PROBLEM 4.64

Air enters a compressor operating at steady state at 14.7 lbf/in.^{2} and 60°F and is compressed to a pressure of 150 lbf/in.^{2}. As the air passes through the compressor, it is cooled at a rate of 10 Btu per lb of air flowing by water circulated through the compressor casing. The volumetric flow rate of the air at the inlet is 5000 ft^3/min, and the power input to the compressor is 700 hp. The air behaves as an ideal gas, there is no stray heat transfer, and kinetic and potential effects are negligible. Determine (a) the mass flow rate of the air, lb/s, and (b) the temperature of the air at the compressor exit, in °F.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas.

ANALYSIS: (a) The mass flow rate is

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 P_1}{RT_1} = \frac{(14.7 \times 144\,\text{lbf/ft}^2)(5000\,\text{ft}^3/\text{min})}{\left(\frac{1545}{28.97}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(520\,°\text{R})}\left|\frac{60\,\text{s}}{1\,\text{min}}\right|$$

$$= 6.36\,\frac{\text{lb}}{\text{s}} \qquad \text{(a)}$$

(b) Reducing an energy rate balance,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \qquad h_2 = h_1 + \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}}$$

With h_1 from Table A-22E and other known data,

$$h_2 = 124.27\,\frac{\text{Btu}}{\text{lb}} - 10\,\frac{\text{Btu}}{\text{lb}} - \frac{[-700\,\text{hp}]}{6.36\,\text{lb/s}}\left|\frac{2545\,\text{Btu/h}}{1\,\text{hp}}\right|\left|\frac{1\,\text{h}}{3600\,\text{s}}\right|$$

$$= 192.08\,\text{Btu/lb}$$

Interpolation in Table A-22E gives

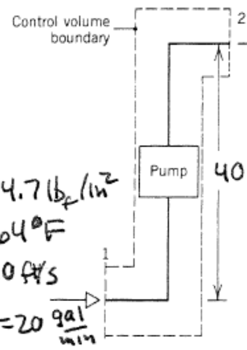
$$T_2 = 801\,°\text{R}\ (341\,°\text{F}) \qquad \text{(b)}$$

PROBLEM 4.65

KNOWN: Water is steadily pumped through a piping arrangement to a higher elevation where it is discharged with a known mass flow rate.

FIND: Determine the power required by the pump.

SCHEMATIC & GIVEN DATA:



$P_2 = 35\ lb_f/in^2$
$T_2 = 64°F$
$V_2 = 40\ ft/s$

$P_1 = 14.7\ lb_f/in^2$
$T_1 = 64°F$
$V_1 = 10\ ft/s$
$(AV)_1 = 20\ \frac{gal}{min}$

$g = 32.0\ \frac{ft}{s^2}$

ENGR. MODEL:
1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} \sim 0$.
3. The water is modeled as incompressible.
4. $g = 32.0\ ft/s^2$

ANALYSIS: Reducing mass and energy rate balances

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right]$$

$$\Rightarrow \quad (-\dot{W}_{cv}) = \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

With assumption 3 and Eq. 3.20b

① $$(h_2 - h_1) = c(\cancel{T_2 - T_1}) + v(P_2 - P_1) = v(P_2 - P_1)$$
$$= \left(0.01604\ \frac{ft^3}{lb}\right)(35 - 14.7)\ \frac{lbf}{in^2}\left|\frac{144\ in^2}{ft^2}\right|\left|\frac{1\ Btu}{778\ ft{\cdot}lbf}\right| = 0.060\ \frac{Btu}{lb}$$

where $v = v_f(64°F)$ from Table A-2E. The change in specific kinetic energy is

$$\frac{V_2^2 - V_1^2}{2} = \left[\frac{(40\ ft/s)^2 - (10\ ft/s)^2}{2}\right]\left|\frac{1\ lbf}{32.2\ lb{\cdot}ft/s^2}\right|\left|\frac{1\ Btu}{778\ ft{\cdot}lbf}\right| = 0.0299\ \frac{Btu}{lb}$$

and

$$g(z_2 - z_1) = \left(32.0\ \frac{ft}{s^2}\right)(40\ ft)\left|\frac{1\ lbf}{32.2\ lb{\cdot}ft/s^2}\right|\left|\frac{1\ Btu}{778\ ft{\cdot}lbf}\right| = 0.0511\ \frac{Btu}{lb}$$

The mass flow rate is calculated using

$$\dot{m} = \frac{(AV)_1}{v} = \frac{(20\ gal/min)}{(0.01604\ ft^3/lb)}\left|\frac{0.13368\ ft^3}{1\ gal}\right|\left|\frac{1\ min}{60\ sec}\right| = 2.78\ lb/s$$

Collecting results

$$(-\dot{W}_{cv}) = \left(2.78\ \frac{lb}{s}\right)\left[0.060 + 0.0299 + 0.0511\right]\ \frac{Btu}{lb}$$
$$= 0.392\ Btu/s$$
$$= \left(0.392\ \frac{Btu}{s}\right)\left|\frac{3600\ s}{1\ h}\right|\left|\frac{1\ hp}{2545\ Btu/h}\right| = 0.55\ hp$$

$(-\dot{W}_{cv})$

1. Alternatively, Eq. 3.13 can be used to evaluate $(h_2 - h_1)$.

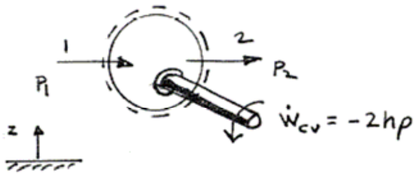
PROBLEM 4.66

KNOWN: Data are provided for a water pump operating at steady state

FIND: Plot the pressure rise from inlet to exit versus volumetric flow.

SCHEMATIC & GIVEN DATA:

$T = 70°F$
$D = 3 in.$
$4 \le AV \le 5$ gal/s
$z_1 = z_2$



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} \approx 0$. 3. Water is modeled as incompressible. (1)

ANALYSIS: The mass rate balance is $\dot{m}_2 = \dot{m}_1$. Since the inlet and exit pipe diameters are the same and the water is incompressible, $V_1 = V_2$; and so the kinetic term vanishes in the energy rate balance, which reads

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + g\cancel{(z_1 - z_2)}\right]$$

(no elevation change)

(2) With Eq. 3.20b,

$$h_2 - h_1 = c\cancel{(T_2 - T_1)}^{0} + v(p_2 - p_1)$$

Thus

$$(-\dot{W}_{cv}) = \dot{m}\left[v(p_2 - p_1)\right] \Rightarrow p_2 - p_1 = \frac{(-\dot{W}_{cv})}{\dot{m}v}$$

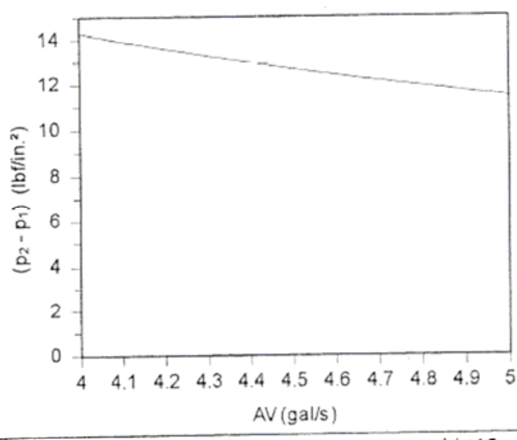
with $\dot{m} = \frac{AV}{v} \Rightarrow AV = \dot{m}v$,

$$(p_2 - p_1) = \frac{(-\dot{W}_{cv})}{(AV)}$$

$$= \frac{(2\,hp)\left|\frac{2545\ Btu/h}{1\ hp}\right|\left|\frac{1\,h}{3600\,s}\right|\left|\frac{778\ ft\cdot lbf}{1\ Btu}\right|\left|\frac{1\ ft^2}{144\ in^2}\right|}{(AV)(gal/s)\left|\frac{0.13368\ ft^3}{1\ gal}\right|}$$

$$= \frac{57.14}{(AV)}\ \frac{lbf}{in^2} \qquad (*)$$

Eq. (*) can be plotted readily using a spreadsheet, plotting program, or IT. The IT plot is shown at the right.



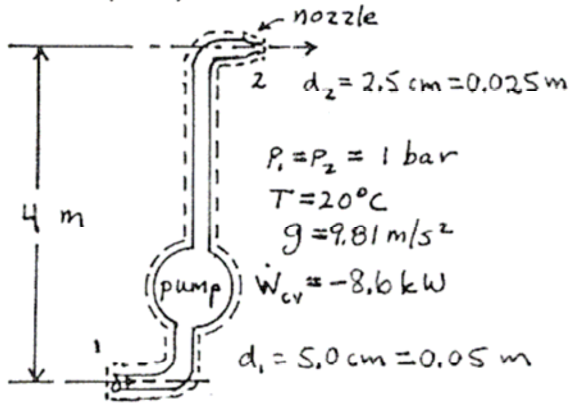
1. Since the water is at 70°F, only a small temperature difference with normal surroundings would be observed. So, heat transfer can be ignored.
2. Alternatively, Eq. 3.13 can be used to obtain the same result.

PROBLEM 4.67

KNOWN: Water flows through pumping system with known inlet and exit conditions. The power required by the pump is also specified.

FIND: Determine the mass flow rate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) The water behaves as an incompessible liquid. (4) The acceleration of gravity is constant at $g = 9.81$ m/s². (5) The temperature and pressure are nearly constant throughout.

ANALYSIS: To find $\dot{m}$, begin with steady-state mass and energy rate balances

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} + \dot{m}\left[\cancel{(h_1 - h_2)}^{0} + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$
$$= -\dot{W}_{cv} + \dot{m}\left[\left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right] \qquad (*)$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$, and the specific enthalpy term is eliminated based on
(1) assumption (3) and Eq. 3.20b.

From Eq. 5.3a, $V = \dot{m}v/A$, and $(*)$ becomes

$$0 = -\dot{W}_{cv} + \dot{m}\left[\frac{(\dot{m}v/A_1)^2 - (\dot{m}v/A_2)^2}{2} + g(z_1 - z_2)\right]$$
$$= -\dot{W}_{cv} + \frac{\dot{m}^3 v^2}{2}\left(\frac{1}{A_1^2} - \frac{1}{A_2^2}\right) + \dot{m}g(z_1 - z_2) \qquad (**)$$

From $A = \pi d^2/4$; $A_1 = 0.000491$ m² and $A_2 = 0.001964$ m². Now, with $v \approx v_{f@20°C} = 1.0018 \times 10^{-3}$ m³/kg from Table A-2, we can insert values in $(**)$

$$0 = -(-8.6\text{ kW})\left|\frac{1\text{ kJ/s}}{1\text{ kW}}\right| + \dot{m}^3\frac{\left(1.0018\times10^{-3}\frac{\text{m}^3}{\text{kg}}\right)^2}{2}\left[\frac{1}{0.000491^2} - \frac{1}{0.001964^2}\right]\frac{1}{\text{m}^4}$$
$$\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| + \dot{m}\left(9.81\frac{\text{m}}{\text{s}^2}\right)(-4\text{ m})\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

or

$$0 = 8.6 - 1.9513\times10^{-3}\,\dot{m}^3 - 0.03924\,\dot{m} \quad (\text{where } \dot{m} \text{ is in kg/s})$$

This equation is cubic in $\dot{m}$. The solution is

$$\dot{m} = 15.98\text{ kg/s} \qquad \dot{m}$$

1. Eq. 3.20b: $h_1 - h_2 = c(T_1 - T_2) + v(P_1 - P_2)$. Here, $T_1 = T_2$, $P_1 = P_2$; so $(h_1 - h_2) = 0$.

PROBLEM 4.68

As shown in Fig. P4.68, a power washer used to clean the siding of a house has water entering through a hose at 20°C, 1 atm and a velocity of 0.2 m/s. A jet of water exits with a velocity of 20 m/s at an average elevation of 5 m with no significant change in temperature or pressure. At steady state, the magnitude of the heat transfer rate *from* the power washer to the surroundings is 10% of the electrical power *input*. Evaluating electricity at 8 cents per kW · h, determine the cost of the power required, in cents per liter of water delivered. Compare with the cost of water, assuming 0.05 cents per liter, and comment.

SCHEMATIC & GIVEN DATA:

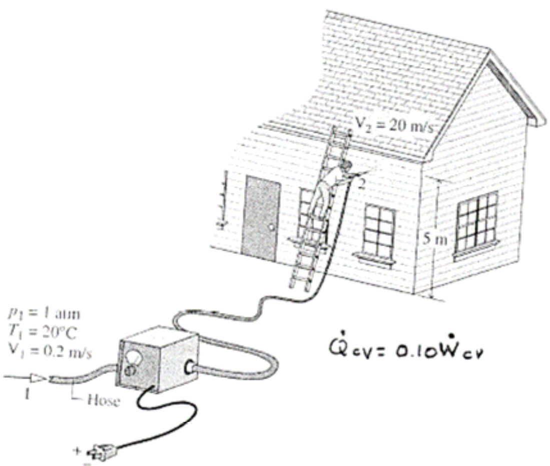


Fig. 4.68

ENGR. MODEL:

1. A control volume encloses the power washer, including the inlet and delivery hoses.
2. The control volume is at steady state.
3. For liquid water, $v \approx v_f(T)$ and $h \approx h_f(T)$.
4. There is no significant change in temperature or pressure from inlet to exit.
5. The cost of electricity is 8 cents per kW·h. The cost of water is 0.05 cents per liter.
6. $g = 9.8\ m/s^2$.

ANALYSIS: Reducing an energy rate balance,

$$0 = \underbrace{\dot{Q}_{cv}}_{=0.1\dot{W}_{cv}} - \dot{W}_{cv} + \underbrace{\dot{m}}_{=\frac{(AV)_1}{v_1}}\left[\underbrace{(h_1-h_2)}_{\approx 0 \text{ by assumptions 3 and 4}} + \left(\frac{V_1^2-V_2^2}{2}\right) + g(z_1-z_2)\right]$$

$$\Rightarrow \quad \frac{\dot{W}_{cv}}{(AV)_1} = \frac{1}{0.9\,v_1}\left[\left(\frac{V_1^2-V_2^2}{2}\right) + g(z_1-z_2)\right]$$

Inserting data, including $v_1 \approx v_f(20°C) = 1.0018 \times 10^{-3}\ m^3/kg$ from Table A-2,

$$\frac{\dot{W}_{cv}}{(AV)_1} = \frac{10^3}{0.9\,(1.0018)\ m^3/kg}\left[\frac{(0.2)^2-(20)^2}{2}\left(\frac{m}{s}\right)^2 + 9.8\frac{m}{s^2}(-5\,m)\right]\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= -276\ \frac{kJ}{m^3}\left|\frac{1\ h}{3600\ s}\right|\left|\frac{1\ kW}{1\ kJ/s}\right|\left|\frac{10^{-3}\ m^3}{1\ L}\right| = -7.7\times10^{-5}\ \frac{kW\cdot h}{L}$$

Costing:

$$\begin{bmatrix}\text{Electricity}\\ \text{per Liter}\end{bmatrix} = 7.7\times10^{-5}\ \frac{kW\cdot h}{L}\left(\frac{8\ \text{cents}}{kW\cdot h}\right) = 6.2\times10^{-4}\ \frac{\text{cents}}{L} \longleftarrow$$

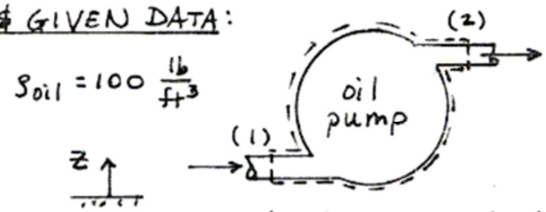
Comment: The cost of the water is significantly greater than the cost of electricity to deliver it: $(0.05\ \text{cents}/L)/(6.2\times10^{-4}\ \text{cents}/L) = 81$.

PROBLEM 4.69

KNOWN: An oil pump operating at steady state delivers oil with a known mass flow rate and pressure rise from inlet to exit.

FIND: If pumps are available in 1/4 - horsepower increments, determine the horsepower rating of the pump needed for this application.

SCHEMATIC & GIVEN DATA:



$\dot{m} = 12$ lb/s
$d_2 = 1$ in. $= (1/12)$ ft
$p_2 - p_1 = 40$ lbf/in.^{2}
T is constant
$z_1 \approx z_2$

ENGR. MODEL: (1) The control volume is at steady state. (2) For the control
1 volume, $\dot{Q}_{cv} \approx 0$. (3) The oil is incompressible. (4) The potential energy change from inlet to exit and the inlet kinetic energy are negligible.

ANALYSIS: To determine the power required, begin with steady state forms of the mass and energy rate balances, as follows:

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{\cancel{V_1^2}^{\,0} - V_2^2}{2}\right) + g\cancel{(z_1 - z_2)}^{\,0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$ and assumptions (2) and (4) have been applied. Thus

$$\dot{W}_{cv} = \dot{m}\left[(h_1 - h_2) - \frac{V_2^2}{2}\right]$$

With Eq. 3.20b

2 $$h_1 - h_2 = c\cancel{(T_1 - T_2)}^{\,0} + v(p_1 - p_2) = \frac{p_1 - p_2}{\rho}$$

Thus

$$\dot{W}_{cv} = \dot{m}\left[\frac{p_1 - p_2}{\rho} - \frac{V_2^2}{2}\right]$$

Using Eq. 4.4b with $A = \pi d^2/4$

$$V_2 = \frac{4\dot{m}}{\rho \pi d^2} = \frac{4\,(12\ \text{lb/s})}{(100\ \text{lb/ft}^3)\,\pi\,(1/12)^2\ \text{ft}^2} = 22\ \text{ft/s}$$

Inserting values and noting that $p_1 - p_2 = -(p_2 - p_1)$

$$\dot{W}_{cv} = \left(12\ \frac{\text{lb}}{\text{s}}\right)\left[\frac{-40\ \text{lbf/in.}^2}{100\ \text{lb/ft}^3}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| - \frac{(22\ \text{ft/s})^2}{2}\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\right]$$

$$= \left(-781.4\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}}\right)\left|\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lbf/s}}\right| = -1.42\ \text{hp}$$

pump size = 1.5 hp ← hp rating

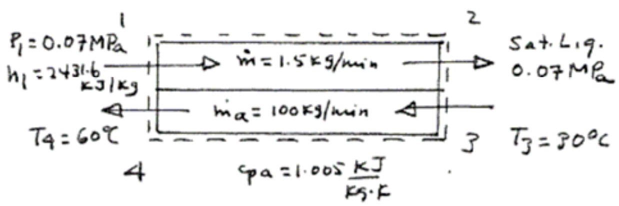
---

1. It is assumed that there is a small temperature difference between the pump and the surroundings, so heat transfer can be ignored.
2. Alternatively, Eq. 3.13 can be used to obtain the same result.

PROBLEM 4.70

Steam enters a heat exchanger operating at steady state at 0.07 MPa with a specific enthalpy of 2431.6 kJ/kg and exits at the same pressure as saturated liquid. The steam mass flow rate is 1.5 kg/min. A separate stream of air with a mass flow rate of 100 kg/min enters at 30°C and exits at 60°C. The ideal gas model with $c_p = 1.005$ kJ/kg · K can be assumed for air. Kinetic and potential energy effects are negligible. Determine (a) the quality of the entering steam and (b) the rate of heat transfer between the heat exchanger and its surroundings, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and all kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas with constant c_{pa}.

ANALYSIS:

(a) From Table A-3 at 0.07 MPa, $h_f = 376.70$ kJ/kg, $h_g = 2660.0$ kJ/kg. Thus, the steam is a two-phase liquid-vapor mixture at 1. Then,

$$x_1 = \frac{h_1 - h_f}{h_g - h_f} = \frac{2431.6 - 376.70}{2660.0 - 376.70} = 0.9$$

(b) Reducing Eq. 4.18,

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right] + \dot{m}_a\left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2}} + \cancel{g(z_3 - z_4)}\right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m}[h_2 - h_1] + \dot{m}_a[\underbrace{h_4 - h_3}_{c_{pa}(T_4 - T_3)}]$$

$$= \dot{m}[h_2 - h_1] + \dot{m}_a c_{pa}(T_4 - T_3)$$

Inserting values,

$$\dot{Q}_{cv} = \left(1.5 \frac{kg}{min}\right)\left[376.7 - 2431.6\right]\frac{kJ}{kg} + \left(100 \frac{kg}{min}\right)\left(1.005 \frac{kJ}{kg \cdot K}\right)(+30 K)$$

$$= -3082.4 \frac{kJ}{min} + 3015 \frac{kJ}{min}$$

$$= -67.35 \frac{kJ}{min}\left|\frac{1 \text{ min}}{60 s}\right|\left|\frac{1 kW}{1 kJ/s}\right|$$

$$= -1.12 \text{ kW}$$

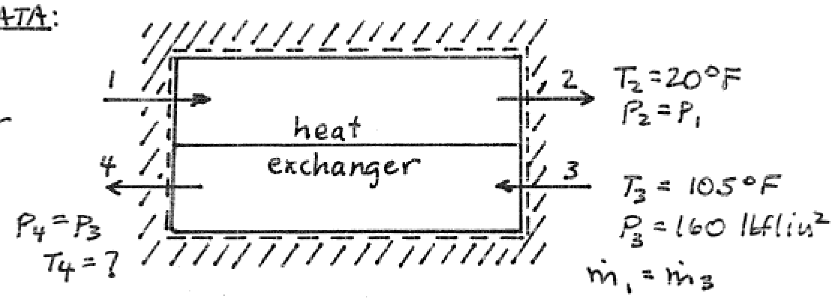
PROBLEM 4.71

KNOWN: Separate vapor and liquid streams of Refrigerant 134a pass in counter flow trough a well-insulated heat exchanger. Data are known at the inlets and exits.

FIND: Determine the exit temperature of the liquid stream.

SCHEMATIC & GIVEN DATA:

R-134a
$T_1 = 0°F$
sat. vapor



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer between the heat exchanger and the surroundings can be neglected and $\dot{W}_{cv} = 0$. (3) Kinetic and potential energy changes from inlet to exit are negligible.

ANALYSIS: To fix state 4, begin with steady state mass and energy balances to determine h_4

$$\dot{m}_1 = \dot{m}_2$$
$$\dot{m}_3 = \dot{m}_4$$

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_1\left[(h_1 - h_2) + \cancelto{0}{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \cancelto{0}{g(z_1 - z_2)}\right] + \dot{m}_3\left[(h_3 - h_4) + \cancelto{0}{\left(\frac{V_3^2 - V_4^2}{2}\right)} + \cancelto{0}{g(z_3 - z_4)}\right]$$

with $\dot{m}_1 = \dot{m}_3$

$$h_4 = (h_1 - h_2) + h_3$$

From Table A-10E; $h_1 = 101.75$ Btu/lb and $p_1 = 21.203$ lbf/in². Interpolating in Table A-12E; $h_2 = 105.76$ Btu/lb.

States 3 and 4 are both subcooled liquid states. The following approximations are reasonable

$$h_3 \approx h_{f@T_3}$$
$$h_4 \approx h_{f@T_4}$$

with $h_3 = 46.01$ Btu/lb from Table A-10E

$$h_4 = (101.75 - 105.76) + 46.01 = 42 \text{ Btu/lb}$$

Interpolating in Table A-10E

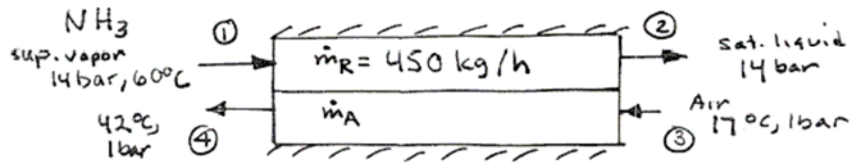
$$T_4 \approx 93.7\ °F \longleftarrow T_4$$

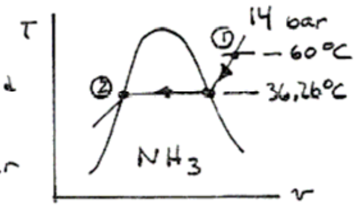
PROBLEM 4.72

KNOWN: Ammonia and air pass in separate streams through a heat exchanger at steady state, for which data are provided.

FIND: Determine the mass flow rate of the air.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: 1. A control volume enclosing the heat exchanger is at steady state. 2. For the control volume, $\dot{W}_{cv} = 0$, heat transfer can be ignored, and kinetic/potential energy effects are negligible.
① 3. Air is modeled as an ideal gas.

ANALYSIS: Since the streams flow separately, the conservation of mass principle indicates at steady state: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_R$ and $\dot{m}_3 = \dot{m}_4 \equiv \dot{m}_A$. An energy rate balance reads

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_R\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right] + \dot{m}_A\left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2}} + \cancel{g(z_3 - z_4)}\right]$$

$\Rightarrow$

$$\dot{m}_A = \dot{m}_R\left[\frac{h_1 - h_2}{h_4 - h_3}\right]$$

From Table A-15, $h_1 = 1542.89$ kJ/kg. From Table A-14, $h_2 = 352.97$ kJ/kg. From Table A-22, $h_3 = 290.16$ kJ/kg, $h_4 = 315.27$ kJ/kg. Then

$$\dot{m}_A = (450)\frac{\text{kg}}{\text{h}}\left[\frac{1542.89 - 352.97}{315.27 - 290.16}\right]\left|\frac{1\ \text{h}}{60\ \text{min}}\right| = 355.4\ \frac{\text{kg}}{\text{min}} \longleftarrow \dot{m}_A$$

---

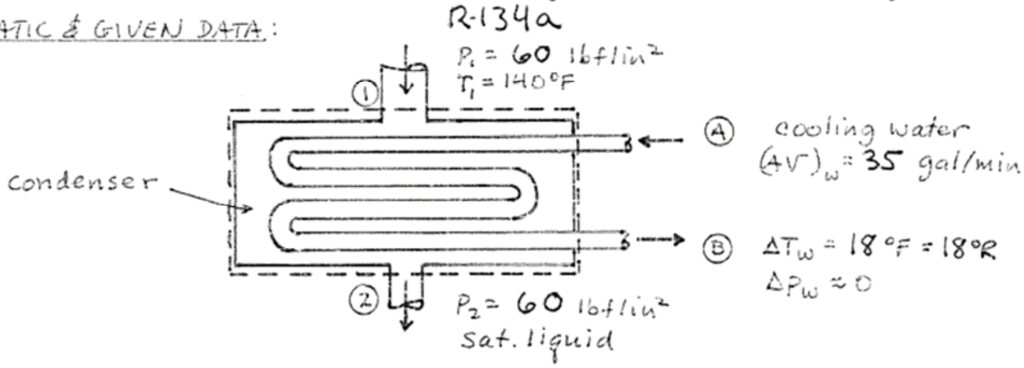
1. The validity of the ideal gas model is readily checked using the generalized compressibility chart.

PROBLEM 4.73

KNOWN: Refrigerant 134a and cooling water pass in separate streams through a condenser (heat exchanger). The volumetric flow rate of cooling water and other data are given at the inlets and exits.

FIND: Determine (a) the mass flow rate of R-134a, and (b) the rate of energy transfer from the condensing R-134a to the cooling water.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer from the outside of the condenser is negligible. (3) Kinetic and potential energy changes from inlet to exit are negligible. (4) The cooling water is modeled as an incompressible liquid with constant specific heat.

ANALYSIS: (a) Since the R-134a and cooling water are separate streams

R-134a: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_R$

Water: $\dot{m}_A = \dot{m}_B \equiv \dot{m}_w$

The mass flow rate of R-134a is found from the steady-state energy balance

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{0} + \cancel{g(z_1 - z_2)}^{0}\right] + \dot{m}_w\left[(h_A - h_B) + \cancel{\left(\frac{V_A^2 - V_B^2}{2}\right)}^{0} + \cancel{g(z_A - z_B)}^{0}\right]$$

Or

$$\dot{m}_R = \frac{\dot{m}_w (h_B - h_A)}{(h_1 - h_2)}$$

For the water, using Eq. 3.20b

$$h_B - h_A = c\,(T_B - T_A) + v\,\cancel{(P_B - P_A)}^{0}$$

and

$$\dot{m}_w = \frac{(AV)_w}{v_w}$$

From Table A-19E; $c \approx 1$ Btu/lb·°R, and $v_w = 1/\rho = 0.0161$ ft³/lb.
For the R-134a, $h_1 = 129.53$ Btu/lb from Table A-12E and $h_2 = 27.24$ Btu/lb from Table A-11E. Inserting values

$$\dot{m}_R = \left(\frac{(35\,\text{gal/min})\left|\dfrac{0.13368\ \text{ft}^3}{1\ \text{gal}}\right|}{(0.0161\ \text{ft}^3/\text{lb})}\right)\frac{(1\ \text{Btu/lb}\cdot°\text{R})(18\ °\text{R})}{(129.53 - 27.24)\ \text{Btu/lb}}\left|\frac{60\ \text{min}}{1\ \text{h}}\right|$$

$= 3068.3$ lb/h ← $\dot{m}_R$

Continued on next slide

Problem 4-73 continued

(b) For a control volume enclosing only the R-134 a

$$0 = \dot{Q}_R - \cancel{\dot{W}_R}^{\,0} + \dot{m}_R \left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{\,0} + g\,\cancel{(z_1 - z_2)}^{\,0} \right]$$

where $\dot{Q}_R$ denotes the heat transfer rate for the R-134 a only. Thus

$$\dot{Q}_R = \dot{m}_R (h_2 - h_1)$$

$$= (3068.3 \text{ lb/h})(27.24 - 129.53)$$

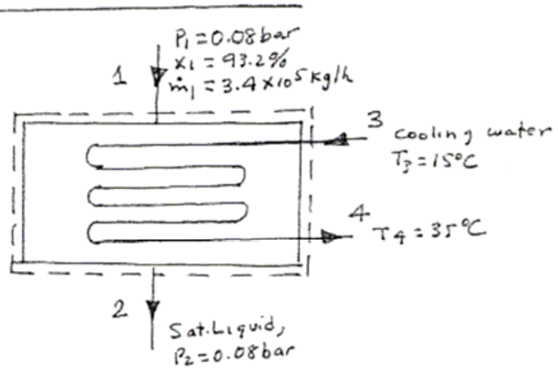
① $$= -3.140 \times 10^5 \text{ Btu/h} \leftarrow \dot{Q}_R$$

1. The negative value for $\dot{Q}_R$ denotes energy transfer heat <u>from</u> the R-134 a to the cooling water, as expected.

PROBLEM 4.74

Steam at a pressure of 0.08 bar and a quality of 93.2% enters a shell-and-tube heat exchanger where it condenses on the outside of tubes through which cooling water flows, exiting as saturated liquid at 0.08 bar. The mass flow rate of the condensing steam is 3.4×10^5 kg/h. Cooling water enters the tubes at 15°C and exits at 35°C with negligible change in pressure. Neglecting stray heat transfer and ignoring kinetic and potential energy effects, determine the mass flow rate of the cooling water, in kg/h, for steady-state operation.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} \approx 0$, and kinetic and potential energy effects can be ignored.
3. For the cooling water, $h \approx h_f(T)$.

ANALYSIS: Reducing Eq. 4.18,

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right] + \dot{m}_3\left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2}} + \cancel{g(z_3 - z_4)}\right]$$

$$\Rightarrow \quad \dot{m}_3 = \dot{m}_1 \frac{[h_1 - h_2]}{[h_4 - h_3]} \qquad (1)$$

where with data from Table A-3,

$$h_1 = \underbrace{h_f}_{= h_2} + x_1(h_g - h_f) = 173.88 + 0.932(2403.1) = 2413.6 \frac{kJ}{kg}.$$

And with data from Table A-2, $h_3 \approx h_f(15°C) = 62.99 \frac{kJ}{kg}$, $h_4 \approx h_f(35°C) = 146.68 \frac{kJ}{kg}$.

Inserting values into Eq. (1), we get

$$\dot{m}_3 = 3.4 \times 10^5 \frac{kg}{h}\left[\frac{2413.6 - 173.88}{146.68 - 62.99}\right]$$

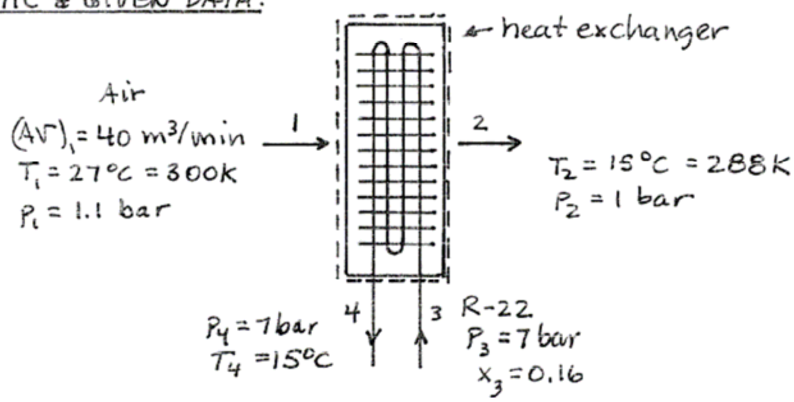
$$= 9.06 \times 10^6 \frac{kg}{h}$$

PROBLEM 4.75

KNOWN: Air and Refrigerant 22 pass in separate streams through a heat exchanger. Data are known at the inlet and exit of each stream.

FIND: Determine (a) the mass flow rate of refrigerant and (b) the rate of energy transfer from the air to the refrigerant.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer from the outside of the heat exchanger is negligible, and $\dot{W}_{cv} = 0$. (3) Kinetic and potential effects can be neglected. (4) The air behaves as an ideal gas, as can be verified by reference to the compressibility chart.

ANALYSIS: (a) The mass flow rate of refrigerant is determined using steady-state mass and energy balances. First, since the air and refrigerant flow as separate streams

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_{air}$$
$$\dot{m}_3 = \dot{m}_4 \equiv \dot{m}_{R-22}$$

Thus, the energy rate balance reduces as follows

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_{air}\left[(h_1 - h_2) + \cancelto{0}{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \cancelto{0}{g(z_1 - z_2)}\right] + \dot{m}_{R-22}\left[(h_3 - h_4) + \cancelto{0}{\left(\frac{V_3^2 - V_4^2}{2}\right)} + \cancelto{0}{g(z_3 - z_4)}\right]$$

and

$$\dot{m}_{R-22} = \dot{m}_{air}\left(\frac{h_1 - h_2}{h_4 - h_3}\right)$$

The mass flow rate of air is found using data at the inlet and the ideal gas equation of state

$$\dot{m}_{air} = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1}$$

$$= \frac{(1.1\ \text{bars})(40\ m^3/min)}{\left(\frac{8.314}{28.97} \cdot \frac{kJ}{kg \cdot K}\right)(300\ K)} \left|\frac{10^5\ N/m^2}{1\ \text{bar}}\right| \left|\frac{1\ kJ}{10^3\ N \cdot m}\right|$$

$$= 51.11\ kg/min$$

Continued on next slide

Problem 4-75 continued

From Table A-22; $h_1 = 300.19$ kJ/kg and $h_2 = 288.15$ kJ/kg. Further, using data from Table A-8

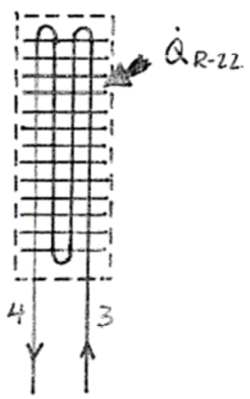
$$h_3 = h_{f3} + x_3 h_{fg3} = 58.04 + (.16)(195.60) = 89.34 \text{ kJ/kg}$$

And, from Table A-9; $h_4 = 256.86$ kJ/kg. Thus

$$\dot{m}_{R\text{-}22} = (51.11 \text{ kg/min})\left(\frac{300.19 - 288.15}{256.86 - 89.34}\right)$$

$$= 3.673 \text{ kg/min} \leftarrow \dot{m}_{R\text{-}22}$$

(b) Consider a control volume enclosing only the refrigerant stream



$$0 = \dot{Q}_{R\text{-}22} + \dot{m}_{R\text{-}22}(h_3 - h_4)$$

$$\dot{Q}_{R\text{-}22} = \dot{m}_{R\text{-}22}(h_4 - h_3)$$

$$= (3.673 \text{ kg/min})(256.86 - 89.34) \text{ kJ/kg}$$

$$= 615.3 \text{ kJ/min} \leftarrow \dot{Q}_{R\text{-}22}$$

<u>COMMENT</u>: For a control volume enclosing only the air stream

$$\dot{Q}_{air} = \dot{m}_{air}(h_2 - h_1)$$

$$= (51.11 \text{ kg/min})(288.15 - 300.19) \text{ kJ/kg}$$

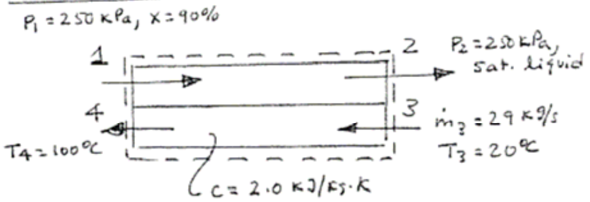
$$= -615.3 \text{ kJ/min}$$

Thus, $\dot{Q}_{R\text{-}22} = -\dot{Q}_{air}$, as expected.

PROBLEM 4.76

Steam enters a heat exchanger operating at steady state at 250 kPa and a quality of 90% and exits as saturated liquid at the same pressure. A separate stream of oil with a mass flow rate of 29 kg/s enters at 20°C and exits at 100°C with no significant change in pressure. The specific heat of the oil is $c = 2.0$ kJ/kg · K. Kinetic and potential energy effects are negligible. If heat transfer from the heat exchanger to its surroundings is 10% of the energy required to increase the temperature of the oil, determine the steam mass flow rate, in kg/s.

SCHEMATIC & GIVEN DATA:



$$\dot{Q}_{cv} = -0.10\underbrace{[\dot{m}_3(h_4-h_3)]}_{\text{energy required to increase the oil temperature}} \quad (1)$$

1

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects are negligible.
3. The oil is modeled as incompressible with $c = 2.0$ kJ/kg·K and no significant change in pressure.

ANALYSIS: Reducing Eq. 4.18,

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)}\right] + \dot{m}_3\left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2} + g(z_3 - z_4)}\right]$$

$$\Rightarrow \quad \dot{m}_1 = \frac{-\dot{Q}_{cv} + \dot{m}_3[h_4 - h_3]}{h_1 - h_2}$$

Introducing Eq. (1),

$$\dot{m}_1 = \frac{+0.10\,\dot{m}_3[h_4 - h_3] + \dot{m}_3[h_4 - h_3]}{h_1 - h_2}$$

$$= \frac{(1.10)\,\dot{m}_3[h_4 - h_3]}{h_1 - h_2} \quad (2)$$

With data from Table A-3, $h_1 = h_f + x_1 h_{fg}$, $h_2 = h_f \Rightarrow h_1 - h_2 = x_1 h_{fg}$. That is, $h_1 - h_2 = 0.90(2181.5\ \text{kJ/kg}) = 1963.4$ kJ/kg.

Inserting values into Eq. (2), using Eq. 3.20b: $(h_4 - h_3) = c(T_4 - T_3) + v\cancel{(p_4 - p_3)}^{\,0}$, we get

$$\dot{m}_1 = \frac{1.10(29\ \text{kg/s})(2.0\ \text{kJ/kg·K})(80\ \text{K})}{1963.4\ \text{kJ/kg}} = 2.6\ \text{kg/s} \longleftarrow$$

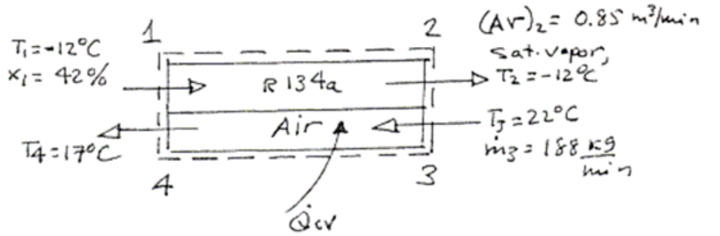
---

1. Consider a control volume consisting of the oil side of the heat exchanger. Assume steady state with $\dot{W}_{cv} = 0$ and ignore kinetic and potential energy effects. An energy rate balance gives $\dot{Q}_{oil} = \dot{m}_3(h_4 - h_3)$, which is the energy that would be required just to increase the oil temperature.

PROBLEM 4.77.

Refrigerant 134a enters a heat exchanger at −12°C and a quality of 42% and exits as saturated vapor at the same temperature with a volumetric flow rate of 0.85 m³/min. A separate stream of air enters at 22°C with a mass flow rate of 188 kg/min and exits at 17°C. Assuming the ideal gas model for air and ignoring kinetic and potential energy effects, determine (a) the mass flow rate of the Refrigerant 134a, in kg/min, and (b) the heat transfer between the heat exchanger and its surroundings, in kJ/min.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential effects are negligib[le]
3. The air is modeled as an ideal gas

ANALYSIS:

$$(a)\quad \dot{m}_2 = \frac{(AV)_2}{v_2} = \frac{0.85\ m^3/min}{0.1068\ m^3/kg} = 7.96\ \frac{kg}{min}\ \ (= \dot{m}_1) \qquad \text{(a)}$$

(Table A-10)

(b) Reducing Eq. 4.18,

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)}\right] + \dot{m}_3\left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2} + g(z_3 - z_4)}\right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m}_1[h_2 - h_1] + \dot{m}_3[h_4 - h_3] \qquad (1)$$

With data from Table A-10, $h_2 = 240.15$ kJ/kg and $h_1 = h_f + x_1 h_{fg} =$ $34.39 + 0.42(205.77) = 120.81$ kJ/kg. Also, from Table A-22, $h_3 = 295.17$ kJ/kg and $h_4 = 290.16$ kJ/kg.

substituting values into Eq. (1), we get

$$\dot{Q}_{cv} = 7.96\frac{kg}{min}(240.15 - 120.81)\frac{kJ}{kg} + 188\frac{kg}{min}(290.16 - 295.17)\frac{kJ}{kg}$$

$$= 949.95\frac{kJ}{min} - 941.88\frac{kJ}{kg}$$

1 $$= +8.1\ \frac{kJ}{kg} \qquad \text{(b)}$$

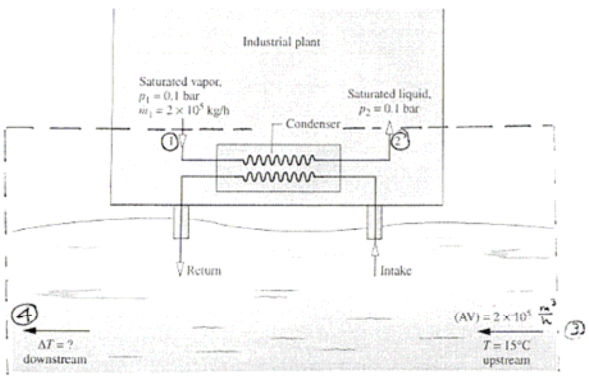
---

1. A positive sign here seems reasonable owing to the low interior temperature of the heat exchanger relative to the temperature of the entering air.

PROBLEM 4.78

As sketched in Fig. P4.78, a condenser using river water to condense steam with a mass flow rate of 2×10^5 kg/h from saturated vapor to saturated liquid at a pressure of 0.1 bar is proposed for an industrial plant. Measurements indicate that several hundred meters upstream of the plant, the river has a volumetric flow rate of 2×10^5 m³/h and a temperature of 15°C. For operation at steady state and ignoring changes in kinetic and potential energy, determine the river-water temperature rise, in °C, downstream of the plant traceable to use of such a condenser, and comment.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$, there is no net heat transfer, and kinetic and potential energy effects can be ignored.
3. At 3 and 4, $v \approx v_f(T)$, $h \approx h_f(T)$.

ANALYSIS: Reducing Eq. 4.18,

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right] + \dot{m}_3\left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2}} + \cancel{g(z_3 - z_4)}\right]$$

$$\Rightarrow \quad h_4 = h_3 + \frac{\dot{m}_1}{\dot{m}_3}\left[h_1 - h_2\right]$$

where

$$\dot{m}_3 = \frac{(AV)_3}{v_3} = \frac{2 \times 10^5 \, m^3/h}{\left(\frac{1.0009}{10^3} \frac{m^3}{kg}\right)} = 2 \times 10^8 \frac{kg}{h}$$

(Table A-2)

Then, with $h_1 - h_2 = h_{fg}$ @ 0.1 bar, $h_1 - h_2 = 2392.8 \frac{kJ}{kg}$ (Table A-3). Also, from Table A-2, $h_3 = 62.99$ kJ/kg.

$$h_4 = 62.99 + \left(\frac{2 \times 10^5}{2 \times 10^8}\right)(2392.8) = 65.38 \frac{kJ}{kg}$$

Interpolation in Table A-3 with $h_4 \approx h_f(T_4)$ gives $T_4 = 15.6°C$

$\Rightarrow$ Temperature rise = 0.6°C

Comment: As discussed in Sec. 2.6.2, adverse environmental consequences can result when large quantities of warm water are returned to a river or lake.

PROBLEM 4.79

Figure P4.79 shows a solar collector panel embedded in a roof. The panel, which has a surface area of 24 ft², receives energy from the sun at a rate of 200 Btu/h per ft² of collector surface. Twenty-five percent of the incoming energy is lost to the surroundings. The remaining energy is used to heat domestic hot water from 90 to 120°F. The water passes through the solar collector with a negligible pressure drop. Neglecting kinetic and potential effects, determine at steady state how many gallons of water at 120°F the collector generates per hour.

SCHEMATIC & GIVEN DATA:

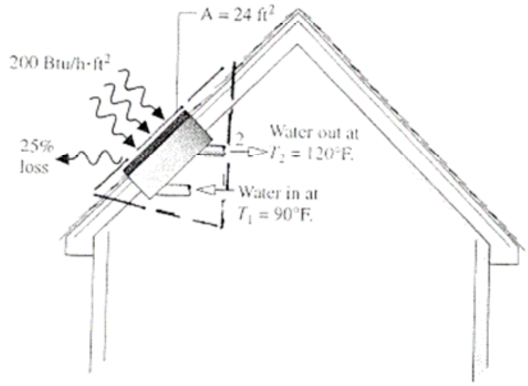


Fig. P4.79

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy can be neglected.
3. For liquid water, $v \approx v_f(T)$ and $h \approx h_f(T)$.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{m} = \frac{\dot{Q}_{cv}}{h_2 - h_1} \qquad (1)$$

where

$$\dot{Q}_{cv} = (0.75)\left(200\ \frac{\text{Btu}}{\text{h}\cdot\text{ft}^2}\right)(24\ \text{ft}^2) = 3600\ \text{Btu/h}$$

Then, with $h_1 \approx h_f(90°F) = 58.07$ Btu/lb, $h_2 \approx h_f(120°F) = 88.0$ Btu/lb from Table A-2E.

Inserting values in Eq.(1),

$$\dot{m} = \frac{3600\ \text{Btu/h}}{(88.0 - 58.07)\ \text{Btu/lb}} = 120.3\ \frac{\text{lb}}{\text{h}}$$

The volumetric flow rate of the water delivered at 120°F is then

$$(AV)_2 = v_2\dot{m} = v_f(120°F)\,\dot{m}$$
$$= \left(0.01621\ \frac{\text{ft}^3}{\text{lb}}\right)\left(120.3\ \frac{\text{lb}}{\text{h}}\right)$$
$$= 1.95\ \text{ft}^3/\text{h}$$

Converting to gallons/h,

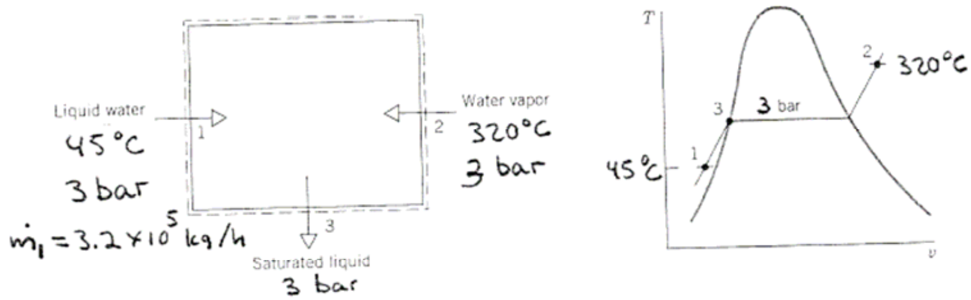
$$(AV)_2 = 1.95\ \frac{\text{ft}^3}{\text{h}}\left|\frac{1\ \text{gal}}{0.13368\ \text{ft}^3}\right| = 14.59\ \frac{\text{gal}}{\text{h}} \longleftarrow$$

PROBLEM 4.80

KNOWN: Data are provided for a feedwater heater at steady state.

FIND: Determine the mass flow rate of the incoming vapor stream, $\dot{m}_2$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$, and heat transfer with the surroundings can be ignored. Kinetic and potential energy effects also can be neglected.

ANALYSIS: At steady state the mass rate balance reads

$$0 = \dot{m}_1 + \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

The energy rate balance reduces to

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left(h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right) + \dot{m}_2\left(h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{gz_2}\right) - \dot{m}_3\left(h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right)$$

$$\Rightarrow \quad 0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

Introducing $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3 \Rightarrow \dot{m}_2 = \dot{m}_1 \left[\frac{h_3 - h_1}{h_2 - h_3}\right]$$

From Table A-4, $h_2 = 3110.1$ kJ/kg. From Table A-3, $h_3 = 561.47$ kJ/kg.
With Eq. 3.14 and data from Table A-2

① $$h_1 \approx h_f(45°C) = 188.45 \text{ kJ/kg}$$

Thus $$\dot{m}_2 = \left(3.2 \times 10^5 \frac{kg}{h}\right)\left[\frac{561.47 - 188.45}{3110.1 - 561.47}\right] = 0.468 \times 10^5 \frac{kg}{h} \longleftarrow \dot{m}_2$$

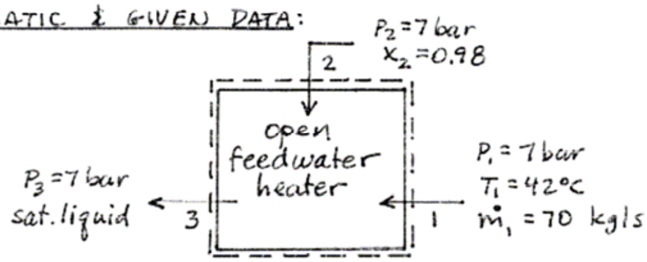
1. Using Eq. 3.13, $h_1 \approx 188.74$ kJ/kg and $\dot{m}_2 = 0.468 \times 10^5$ kg/h

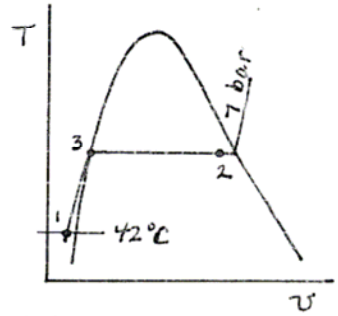
PROBLEM 4.81

KNOWN: An open feedwater heater operates with known inlet and exit conditions. The mass flow rate at one inlet is given.

FIND: Determine the mass flow rate at the second inlet.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer with the surroundings is negligible, and $\dot{W}_{cv} = 0$. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: To find $\dot{m}_2$, begin with the steady-state mass and energy balances

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + \dot{m}_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right) - \dot{m}_3\left(h_3 + \frac{V_3^2}{2} + gz_3\right)$$

With $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$ and assumption (3)

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

or

$$\dot{m}_2 = \dot{m}_1\left(\frac{h_1 - h_3}{h_3 - h_2}\right)$$

From Table A-3, $T_1 < T_{sat}$ at 7 bar. Hence, state 1 is compressed liquid. Using Eq. 3.13 and data from Table A-2 at 42°C

(1)
$$h_1 \approx h_f(T_1) + v_f(T_1)\left[p - p_{sat}(T_1)\right]$$
$$= 175.9 \text{ kJ/kg} + \left(1.0086 \times 10^{-3} \frac{m^3}{kg}\right)\left[7 - 0.08268\right] \text{bar} \left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|$$
$$= 176.6 \text{ kJ/kg}$$

Further, from Table A-3

$$h_2 = h_{f2} + x_2 h_{fg2} = 697.22 + (.98)(2066.3) = 2722.2 \text{ kJ/kg}$$
$$h_3 = 697.22 \text{ kJ/kg}$$

Finally

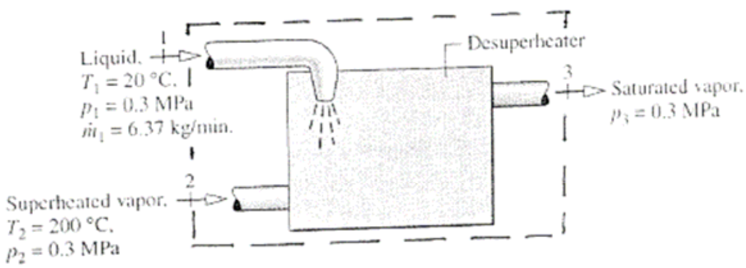
$$\dot{m}_2 = (70 \text{ kg/s})\left(\frac{176.6 - 697.22}{697.22 - 2722.2}\right)$$
$$= 18.0 \text{ kg/s}$$ $\longleftarrow$ $\dot{m}_2$

1. Alternatively, Eq. 3.14 can be used: $h_1 \approx h_f(T_1) = 175.9$ kJ/kg. The result is $\dot{m}_2 = 18.02$ kg/s.

PROBLEM 4.82

For the *desuperheater* shown in Fig. P4.82, liquid water at state 1 is injected into a stream of superheated vapor entering at state 2. As a result, saturated vapor exits at state 3. Data for steady state operation are shown on the figure. Ignoring stray heat transfer and kinetic and potential energy effects, determine the mass flow rate of the incoming superheated vapor, in kg/min.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and potential kinetic and energy effects can be ignored. $\dot{W}_{cv} \equiv 0$.
3. At state 1, $h_1 \approx h_f(T_1)$.

ANALYSIS:

Mass rate balance: $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ (1)

Energy rate balance,

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right] + \dot{m}_2\left[h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{gz_2}\right] - \dot{m}_3\left[h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right]$$

$$\Rightarrow 0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3 \qquad (2)$$

Combining Eqs. (1), (2)

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

$$\Rightarrow \dot{m}_2 = \dot{m}_1 \left[\frac{h_3 - h_1}{h_2 - h_3}\right] \qquad (3)$$

From Table A-2, $h_1 \approx 83.96$ kJ/kg. From Table A-3, $h_3 = 2725.3$ kJ/kg. From Table A-4, $h_2 = 2865.5$ kJ/kg. Inserting values in Eq. (3),

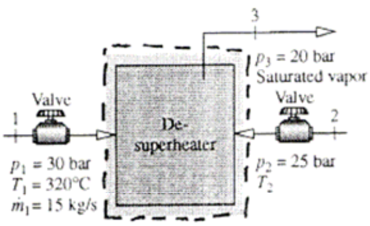
$$\dot{m}_2 = 6.37 \frac{kg}{min}\left[\frac{2725.3 - 83.96}{2865.5 - 2725.3}\right] = 120.01 \frac{kg}{min}$$

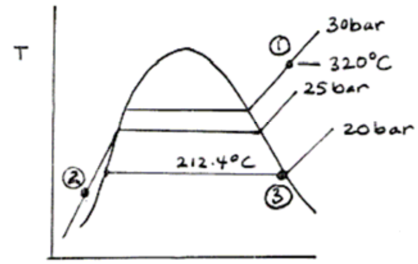
PROBLEM 4.83

KNOWN: Data are provided for a desuperheater operating at steady state.

FIND: (a) For a specified temperature for the entering liquid, determine the liquid mass flow rate. (b) Plot the liquid mass flow rate versus the liquid temperature.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: 1. A control volume enclosing the desuperheater with inlets at 1 and 2 and an exit at 3 is at steady state. 2. For the control volume, $\dot{W}_{cv} = 0$ and heat transfer with the surroundings can be ignored. Kinetic and potential energy effects can be ignored.

ANALYSIS: The mass rate balance at steady state reads $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$. The energy rate balance at steady state reduces as follows:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right] + \dot{m}_2\left[h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{gz_2}\right] - \dot{m}_3\left[h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right]$$

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

Solving for $\dot{m}_2$

$$\dot{m}_2 = \dot{m}_1 \left[\frac{h_3 - h_1}{h_2 - h_3}\right] \qquad (1)$$

(a) From Table A-4; h_1 = 3043.4 kJ/kg. From Table A-3; h_3 = 2799.5 kJ/kg. Further, at T_2 = 200 °C, p_2 = 25 bar; Table A-5 gives h_2 = 852.8 kJ/kg. Thus

$$\dot{m}_2 = \left(15 \frac{kg}{s}\right)\left[\frac{2799.5 - 3043.4}{852.8 - 2799.5}\right] = 1.88 \text{ kg/s} \longleftarrow \dot{m}_2 \text{ (part a)}$$

(b) The following IT code is used to develop data to construct a plot of $\dot{m}_2$ vs. T_2:

<u>IT Code</u>

```
p1 = 30  // bar
T1 = 320  // °C
mdot1 = 15  // kg/s
p2 = 25  // bar
T2 = 20  // °C
p3 = 20  // bar

h1 = h_PT("Water/Steam", p1, T1)
h2 = h_PT("Water/Steam", p2, T2)
h3 = hsat_Px("Water/Steam", p3, 1)

mdot2 = mdot1 * ((h3 - h1) / (h2 - h3))
```

1

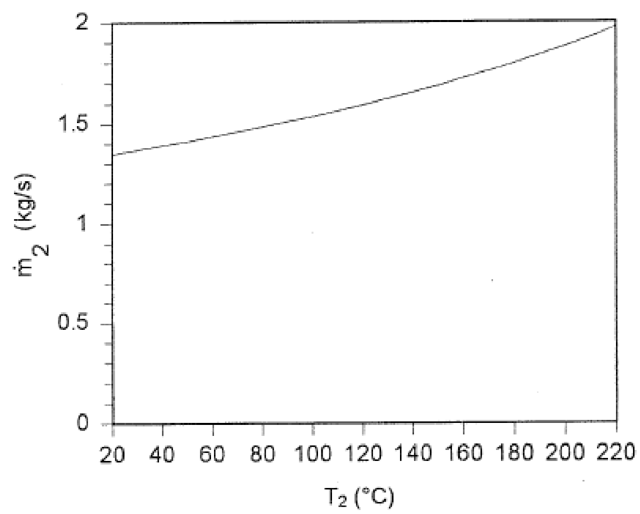
Continued on next slide

Problem 4-83 continued

IT Result for T_2 = 200°C

h_1 = 3043 kJ/kg
h_2 = 852.4 kJ/kg
h_3 = 2799 kJ/kg
mdot2 = 1.879 kg/s

This result compares very favorably with the result of part (a). Thus, with the computer solution validated, use the Explore button and sweep T_2 from 20 to 220°C in steps of 10. The resulting data are used to construct the following plot:

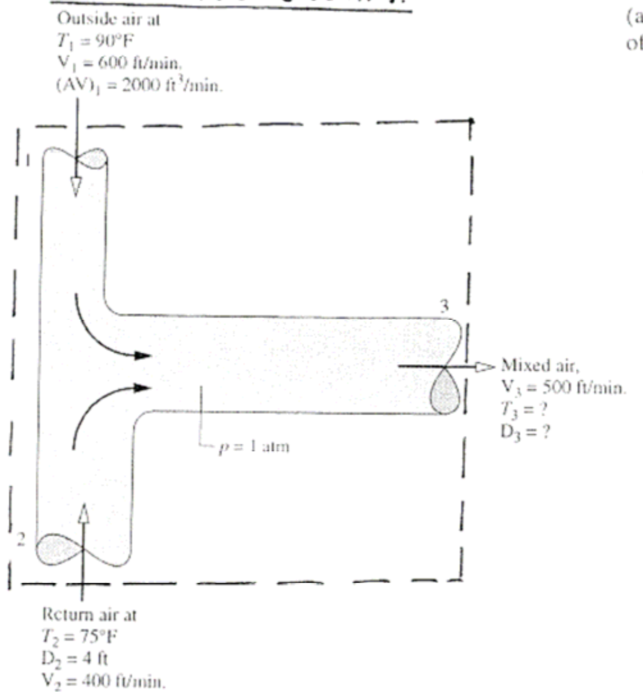


1. Note that IT uses the approximation of Eq. 3.14 for liquid enthalpies.

PROBLEM 4.84

Figure P4.84 provides steady-state data for the ducting ahead of the chiller coils in an air conditioning system. Outside air at 90°F is mixed with return air at 75°F. Stray heat transfer is negligible, kinetic and potential energy effects can be ignored, and the pressure throughout is 1 atm. Modeling the air as an ideal gas with c_p = 0.24 Btu/lb · R, determine (a) the mixed-air temperature, in °F, and (b) the diameter of the mixed-air duct, in ft.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and kinetic and potential energy effects are negligible. $\dot{W}_{cv} = 0$.
3. The air is modeled as an ideal gas with $c_p = 0.24$ Btu/lb.

ANALYSIS: Mass rate balance: $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$, where

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1} = \frac{\left(14.7\times144 \frac{lbf}{ft^2}\right)\left(2000 \frac{ft^3}{min}\right)}{\left(\frac{1545}{28.97}\frac{ft\cdot lbf}{lb\cdot °R}\right)(550°R)} = 144.33 \frac{lb}{min}$$

$$\dot{m}_2 = \frac{A_2 V_2}{v_2} = \frac{\left(\frac{\pi D_2^2}{4}\right)(V_2)}{RT_2/p_2} = \frac{p_2\left(\frac{\pi D_2^2}{4}\right)(V_2)}{\bar{R}\; T_2} = \frac{\left(14.7\times144\frac{lbf}{ft^2}\right)\left(\frac{\pi}{4}(4ft)^2\right)\left(400\frac{ft}{min}\right)}{\left(\frac{1545}{28.97}\frac{ft\cdot lbf}{lb\cdot °R}\right)(535°R)} = 372.92 \frac{lb}{min}$$

(a) Energy rate balance:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left(h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right) + \dot{m}_2\left(h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{gz_2}\right) - (\dot{m}_1+\dot{m}_2)\left(h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right)$$

$$\Rightarrow \quad 0 = \dot{m}_1[h_1 - h_3] + \dot{m}_2[h_2 - h_3] \Rightarrow 0 = \dot{m}_1 c_p (T_1 - T_3] + \dot{m}_2 c_p [T_2 - T_3]$$

$$\Rightarrow \quad T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2} = \frac{(144.33)(550°R) + (372.92)(535°R)}{(144.33 + 372.92)} = 539°R\,(79°F) \quad \longleftarrow (a)$$

(b) $\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 517.25 \frac{lb}{min}$. Also, $\dot{m}_3 = \frac{A_3 V_3}{v_3} = \frac{A_3 V_3\, p_3}{R\, T_3}$ and $A_3 = \frac{\pi D_3^2}{4}$. Collecting results

$$D_3 = \sqrt{\frac{4}{\pi}\frac{\dot{m}_3 R T_3}{p_3 V_3}} = \sqrt{\frac{4}{\pi}\frac{\left(517.25\frac{lb}{min}\right)\left(\frac{1545}{28.97}\frac{ft\cdot lbf}{lb\cdot °R}\right)(539°R)}{(14.7\times144\ lbf/ft^2)(500\ ft/min)}} = 4.23\ ft \quad \longleftarrow (b)$$

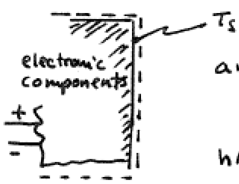
PROBLEM 4.85

KNOWN: The electronic components of Example 4.8 are cooled by air flowing through the electronics enclosure.

FIND: Determine the largest average surface temperature of the components for which specified limits are met.

SCHEMATIC & GIVEN DATA:

Also See Fig. E4.8



air temperature:

$$T_a = \frac{T_1 + T_2}{2}$$

$$hA = 5 \text{ W/K}$$

ENGR. MODEL: 1. The electronic components form the system, which is at steady state.

ANALYSIS: The energy transfer from the electronic components to the air by convection is

$$\dot{Q}_c = hA[T_s - T_a]$$

① At steady state, the electric power provided to the electric components equals the energy removed by heat transfer: $\dot{Q}_c = 80$ W. Also, $hA = 5$ W/K. Solving for T_s, noting that $T_1 = 293$K and $T_2 \leq 305$K (22°C)

$$T_s = T_a + \frac{\dot{Q}_c}{hA}$$

$$T_s = \left(\frac{T_1 + T_2}{2}\right) + \frac{\dot{Q}_c}{hA}$$

② $$T_s \leq \left(\frac{293 + 305}{2}\right)\text{K} + \frac{80 \text{ W}}{5 \text{ W/K}} = 315 \text{ K } (42°\text{C})$$

1. This can be obtained formally by applying an energy rate balance to a system consisting of the electronic components.

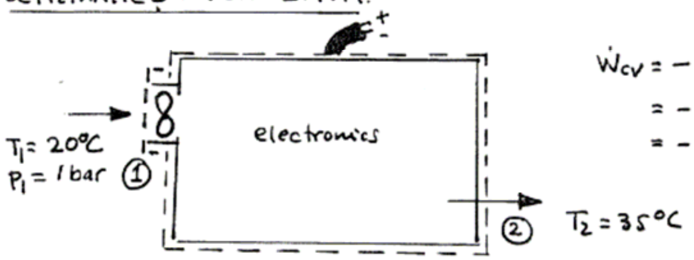
2. For reliability, excessive temperatures within the electronics are avoided by controlling the surface temperature T_s.

PROBLEM 4.86

KNOWN: Data are provided for an electronics enclosure cooled by an air flow induced by a fan.

FIND: Determine the volumetric flow rate of the air entering the fan.

SCHEMATIC & GIVEN DATA:



$$\dot{W}_{cv} = -[25W + 100W]$$
$$= -125W$$
$$= -0.125\,kW$$

ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, heat transfer with the surroundings and kinetic/potential energy effects can be ignored. 3. Air is modeled as an ideal gas.

ANALYSIS: At steady state $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. The energy rate balance reads

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

or

$$\dot{m} = \frac{(-\dot{W}_{cv})}{h_2 - h_1}$$

with $\dot{m} = (AV)_1/v_1$ and using the ideal gas equation of state

$$\dot{m} = \frac{(AV)_1 p_1}{RT_1}$$

Collecting results and inserting data, including enthalpy values from Table A-22

$$(AV)_1 = \left(\frac{RT_1}{p_1}\right)\left[\frac{(-\dot{W}_{cv})}{h_2 - h_1}\right]$$

$$= \frac{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(293\,K)}{(10^5\,N/m^2)}\left[\frac{0.125\ kJ/s}{(308.2 - 293.2)\frac{kJ}{kg}}\right]$$

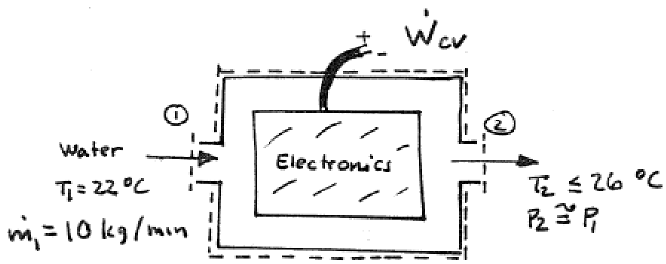
$$= 7 \times 10^{-3}\ \frac{m^3}{s}$$

PROBLEM 4.87

KNOWN: Data are provided for a water-jacketed housing filled with electronic components, which is at steady state.

FIND: Determine the maximum power input to satisfy a limit on the temperature of the water exiting the enclosure.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, heat transfer with the surroundings can be ignored, as can kinetic/potential energy effects. 3. For the water

① entering and exiting the housing, $h \approx h_f(T)$.

ANALYSIS: At steady state, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. An energy rate balance reads

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \cancel{g(z_1 - z_2)}\right]$$

giving

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

$$= \dot{m}\left(h_f(T_1) - h_f(T_2)\right)$$

Since $T_2 \leq 26°C$

$$\dot{W}_{cv} \geq \dot{m}\left(h_f(22°C) - h_f(26°C)\right)$$

$$\geq \left(10\,\frac{kg}{min}\right)(92.33 - 109.07)\frac{kJ}{kg}\left|\frac{1\ min}{60\ s}\right|\left|\frac{1\ kW}{1\ kJ/s}\right|$$

② $$\dot{W}_{cv} \geq -2.79\ kW \longleftarrow$$

---

1. Alternatively, Eq. 3.20b with c from Table A-19 can be used.

2. By the usual sign convention, $\dot{W}_{cv}$ represents power <u>output</u>. Hence, the constraint on power <u>input</u> can be expressed alternatively in terms of the <u>magnitude</u> of $\dot{W}_{cv}$ as

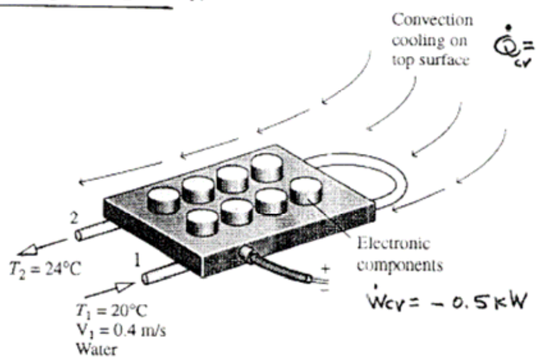
$$0 \leq |\dot{W}_{cv}| \leq 2.79\ kW$$

PROBLEM 4.88

KNOWN: Data are provided for electronic components mounted on a plate that are cooled by convection to the surroundings and water circulating through a tube bonded to the plate. Operation is at steady state.

FIND: Determine the tube diameter.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. A control volume encloses the plate-mounted electronic components with an inlet at 1 and an exit at 2. 2. The control volume is at steady state. 3. For the water entering and exiting, $h \approx h_f(T)$, $v \approx v_f(T)$. (1)
4. Kinetic and potential energy effects can be ignored.

ANALYSIS: At steady state, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Also,

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{(\pi D^2/4) V_1}{v_f(T_1)}$$

An energy rate balance reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{(V_1^2 - V_2^2)}{2}} + \cancel{g(z_1 - z_2)}\right]$$

or

$$\dot{m} = \frac{[\dot{Q}_{cv} - \dot{W}_{cv}]}{h_2 - h_1} = \frac{[\dot{Q}_{cv} - \dot{W}_{cv}]}{h_f(T_2) - h_f(T_1)}$$

Collecting results

$$D = \sqrt{\frac{4\, v_f(T_1)}{\pi V_1}\left[\frac{(\dot{Q}_{cv} - \dot{W}_{cv})}{h_f(T_2) - h_f(T_1)}\right]}$$

with data from Table A-2: $v_f(20°C) = (1.0018/10^3)\ m^3/kg$, $h_f(T_1) = 83.96$ kJ/kg, $h_f(T_2) = 100.7$ kJ/kg

$$D = \sqrt{\frac{(4)(1.0018/10^3)\,m^3/kg}{\pi\,(0.4\,m/s)}\left[\frac{(-0.08 - (-0.5))\,kJ/s}{(100.7 - 83.96)\,kJ/kg}\right]}$$

$$= 0.0089\ m \left|\frac{10^2\,cm}{m}\right|$$

$$= 0.89\ cm$$

1. Alternatively, the incompressible model can be used, with Eq 3.20b and c from Table A-19.

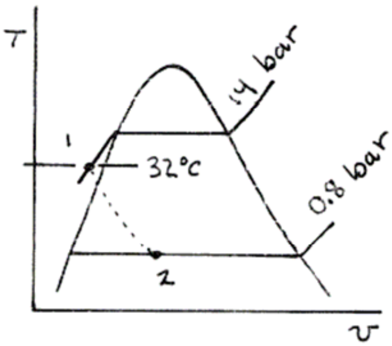
PROBLEM 4.89

KNOWN: Ammonia expands through a valve from a known pressure and temperature to a given final pressure.

FIND: Determine the exit quality.

SCHEMATIC & GIVEN DATA:

$p_1 = 1.4$ MPa $= 14$ bar, $T_1 = 32$ °C (1) → valve → (2) $p_2 = 0.08$ MPa $= 0.8$ bar, $x_2 = ?$



ENGR. MODEL: (1) A control volume enclosing the valve is at steady state. (2) The refrigerant undergoes a throttling process; $h_1 = h_2$.

ANALYSIS: According to data from Table A-14, at $p_1 = 14$ bar; $T_{sat} = 36.26$ °C. Since $T_1 < T_{sat}$, state 1 is in the compressed liquid region. For simplicity,
① we use Eq. 3.14 to evaluate h_1, as follows:

$$h_1 \approx h_f(T_1) = 332.17 \text{ kJ/kg}$$

where the value is obtained from Table A-13.

By assumption (2)

$$h_1 = h_2$$
$$= h_{f2} + x_2 h_{fg2}$$

Solving and inserting data from Table A-14 at $p_2 = 0.8$ bar

$$x_2 = \frac{h_1 - h_{f2}}{h_{fg2}}$$
$$= \frac{332.17 \text{ kJ/kg} - 9.04 \text{ kJ/kg}}{1382.73 \text{ kJ/kg}}$$
$$= 0.2337 \ (23.37\%) \leftarrow x_2$$

---

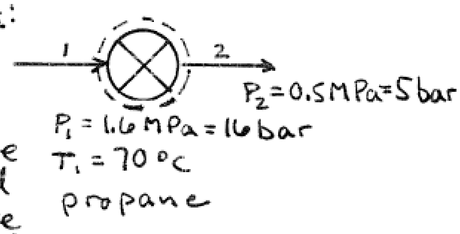
1. Here, we have ignored the effect of pressure on the specific enthalpy of liquid refrigerant. We could have used Eq. 3.13 to estimate the effect of pressure. In that case, h_1 would have been 332.44 kJ/kg, and the exit quality would have been $x_2 = 0.2339$ (23.39%). Thus, using Eq. 3.14 is very accurate in this case, and the approximation of Eq. 3.14 is commonly used when refrigeration systems are analyzed.

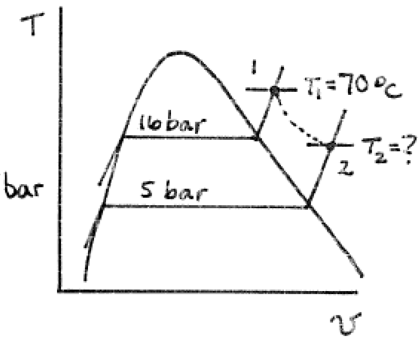
PROBLEM 4.90

KNOWN: Propane expands through a valve from a known inlet state to a known exit pressure.

FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible and $\dot{W}_{cv} = 0$. (3) Kinetic and potential energy effects are negligible. (throttling process)

ANALYSIS: In accordance with the assumptions for a throttling process, $h_1 = h_2$. Using data from Table A-18

$$h_2 = h_1 = 568.5 \quad kJ/kg$$

Interpolating in Table A-18 at $p_2 = 5$ bar, $h_2 = 568.5$ kJ/kg

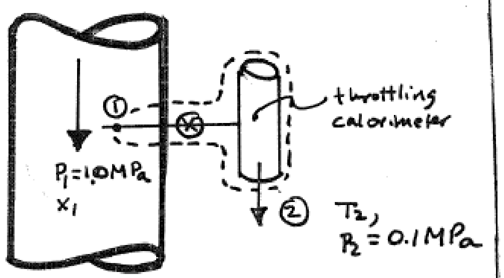
$$T_2 \approx 54.7\ °C \longleftarrow T_2$$

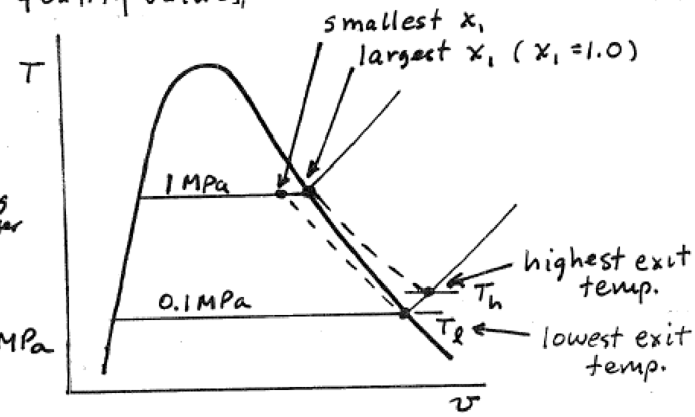
PROBLEM 4.91

KNOWN: Data are provided for a throttling calorimeter attached to a large pipe carrying a two-phase liquid–vapor mixture. Operation is at steady state. The substance is H_2O.

FIND: Determine the range of throttling calorimeter exit temperatures for which the device can determine the quality of steam in the pipe, and the corresponding range of steam quality values.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: 1. The control volume shown in the figure is at steady state. 2. The expansion through the calorimeter adheres to the throttling process model: $h_2 \approx h_1$, where $h_1 = h_{f1} + x_1(h_{g1} - h_{f1})$.

ANALYSIS: With assumption 2

$$h(T_2, P_2) = h_{f1} + x_1(h_{g1} - h_{f1}) \qquad (1)$$

where the state at the exit is fixed by T_2, P_2, and thus must be superheated vapor or, in the limit, saturated vapor. Also, the quality x_1 is required to be less than or, in the limit, equal to 1.0.

Referring to the T-v diagram, the highest exit temperature, T_h, corresponds to a steam quality of 1.0: That is, with data from Table A-3, Eq. (1) gives

$$h(T_h, P_2) = h_g(P_1)$$
$$= 2778.1 \text{ kJ/kg}$$

Interpolating in Table A-4 at 0.1 MPa gives $T_h = 151°C$.

The lowest exit temperature, T_ℓ, corresponds to saturated vapor exiting the calorimeter, for which $h_2 = 2675.5$ kJ/kg and $T_\ell = 99.6°C$. Eq. (1) gives

$$x_1 = \frac{h_2 - h_{f1}}{h_{g1} - h_{f1}} = \frac{2675.5 - 762.81}{2015.3} = 0.949$$

In summary

$$99.6 \le T_2 \le 151 \text{ °C}$$
$$0.949 \le x_1 \le 1.0$$

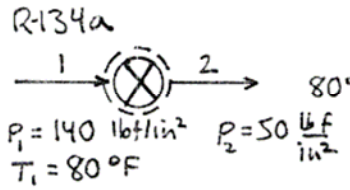
PROBLEM 4.92

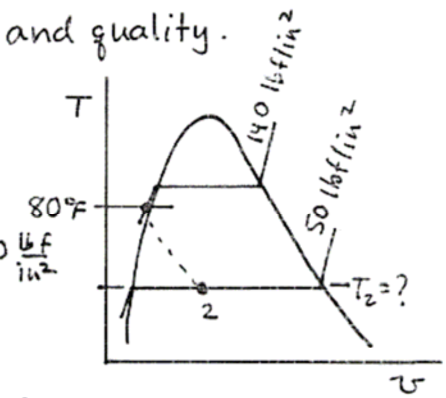
KNOWN: Refrigerant 134a expands through a valve from a known inlet state to a known final pressure.

FIND: Determine the exit temperature and quality.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible, and $\dot{W}_{cv} = 0$. (3) Kinetic and potential energy effects are negligible. (throttling process)





ANALYSIS: In accordance with the assumptions for a throttling process, $h_1 = h_2$. From Table A-11E; $T_1 < T_{sat}$ @ 140 lbf/in². Thus, the inlet state is in the compressed liquid region. Ignoring the effect of pressure on the enthalpy of the liquid, Table A-10E gives

$$h_1 \approx h_f(80°F) = 37.27 \text{ Btu/lb}$$

Since $h_2 = h_1 = h_{f2} + x_2 h_{fg2}$, data from Table A-11E at 50 lbf/in² give

$$x_2 = \frac{h_2 - h_{f2}}{h_{fg2}}$$

$$= \frac{37.27 - 24.14}{83.29}$$

$$= 0.158 \quad (15.8\%) \longleftarrow x_2$$

and

$$T_2 = 40.27°F \longleftarrow T_2$$

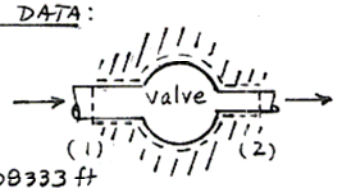
PROBLEM 4.93

KNOWN: Steam flows through a well-insulated valve from specified inlet conditions to a known exit pressure.

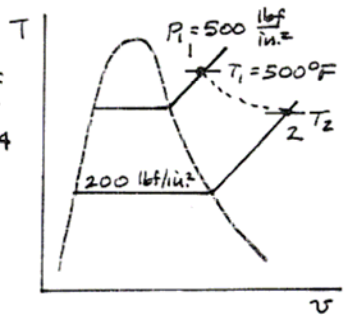
FIND: (a) Determine the exit velocity and exit temperature for a given ratio of inlet-to-exit pipe diameters, d_1/d_2. (b) Plot the exit velocity, temperature, and specific enthalpy versus d_1/d_2 ranging from 0.25 to 4.

SCHEMATIC & GIVEN DATA:

Steam
$p_1 = 500\ \frac{\text{lbf}}{\text{in.}^2}$
$T_1 = 500°F$
$d_1 = 1\text{ in.} = 0.08333\text{ ft}$



$p_2 = 200\ \frac{\text{lbf}}{\text{in}^2}$
$0.25 \le \frac{d_1}{d_2} \le 4$
$\dot{m} = 0.11\text{ lb/s}$



ENGR. MODEL: (1) The control volume is at steady state.
(2) For the control volume, $\dot{Q}_{cv} = 0$ and $\dot{W}_{cv} = 0$.
(3) Potential energy effects are negligible.

ANALYSIS: (a) To determine the exit velocity, begin with the mass balance and Eq. 4.4b: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$, and

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2} \Rightarrow V_2 = \frac{A_1}{A_2}\cdot\frac{V_1}{v_1}\cdot v_2 = \left(\frac{d_1}{d_2}\right)^2 \cdot \frac{V_1}{v_1}\cdot v_2$$

Now, $V_1/v_1 = \dot{m}/A_1 = 4\dot{m}/\pi d_1^2$. Inserting values

$$\frac{V_1}{v_1} = \frac{(4)(.11\text{ lb/s})}{\pi(.08333\text{ ft})^2} = 20.17$$

Thus

$$V_2 = 20.17\ \left(\frac{d_1}{d_2}\right)^2 \cdot v(T_2, p_2) \qquad (1)$$

and, with $v_1 = 0.992\text{ ft}^3/\text{lb}$ from Table A-4E

$$V_1 = (20.17)(0.992) = 20.01\text{ ft/s}$$

From (1) we see that it is necessary to fix state 2 to evaluate V_2. Another relation is obtained from the energy balance at steady state.

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancel{(z_1 - z_2)}^{\,0}\right] \qquad (2)$$

or

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2}$$

with h_1 from Table A-4E

$$V_2 = \sqrt{2\left[1231.5 - h(T_2, p_2)\right]\frac{\text{Btu}}{\text{lb}}\left|\frac{1\text{ lbf}}{32.2\text{ lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| + (20.01\text{ ft/s})^2} \qquad (3)$$

Equations (1) and (3) can be solved simultaneously using data from Table A-4E and an iterative process. The results are, for $d_1/d_2 = 0.25$

$T_2 = 434.6°F$ ◄

$V_2 = 3.140\text{ ft/s}$ ◄

Continued on next slide

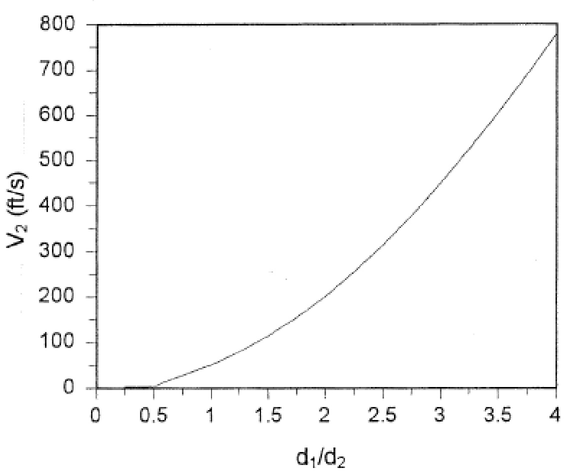
(b) The following *IT* code can be used to solve Eqs. (1) and (2) with the associated data for steam:

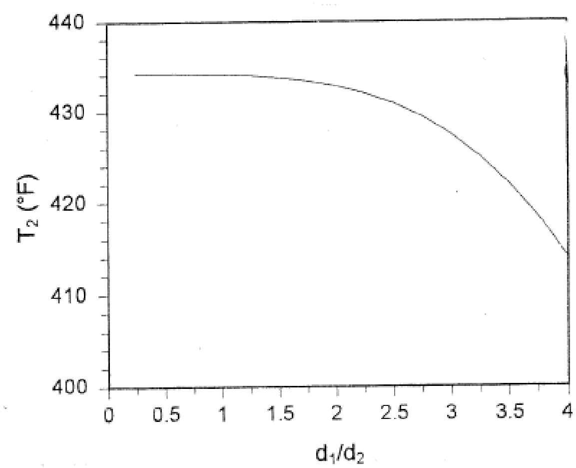
```
p1 = 500  // lbf/in.²
T1 = 500  // °F
mdot = 0.11  // lb/s
d1 = 1/12  // ft.
p2 = 200  // lbf/in.²
dratio = d1 / d2
dratio = .25

A1 = pi * d1^2 / 4
mdot = A1 * V1 / v1
V2 / V1 = (d1 / d2)^2 * (v2 / v1)

h1 = h_PT("Water/Steam", p1, T1)
v1 = v_PT("Water/Steam", p1, T1)
h2 = h_PT("Water/Steam", p2, T2)
v2 = v_PT("Water/Steam", p2, T2)

0 = (h1 - h2) + ((V1^2 - V2^2) / (2 * 32.2 * 778))
```

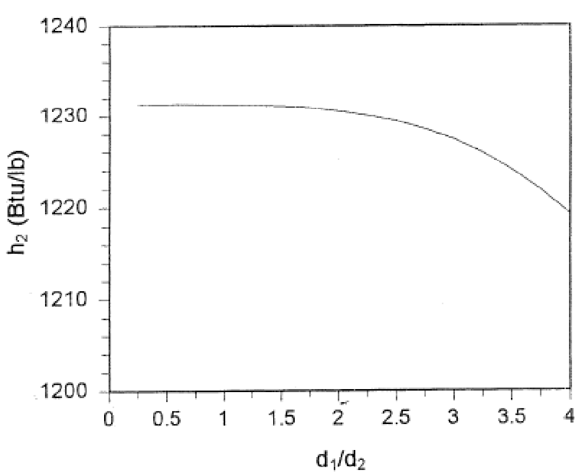
<u>*IT* Results for the sample case of $d_1/d_2 = 0.25$</u>

$T_2 = 434.2$ °F
$V_2 = 3.139$ ft/s
$h_2 = 1231$ Btu/lb
$V_1 = 20.01$ ft/s
$v_2 = 2.490$ ft³/lb

Using the Explore button, sweep dratio from 0.25 to 4 in steps of 0.25. Then, construct the following plots:







COMMENT:

Note that as d_1/d_2 becomes large the throttling process assumptions (Sec. 4.10) of negligible kinetic energy and $h_2 \approx h_1$ are not valid.

PROBLEM 4.94

KNOWN: Steady-state operating data are provided for a turbine powering a compressor and electric generator.

FIND: For the turbine determine (a) the volumetric flow rate of the air at the exit and (b) the heat transfer rate.

SCHEMATIC & GIVEN DATA:

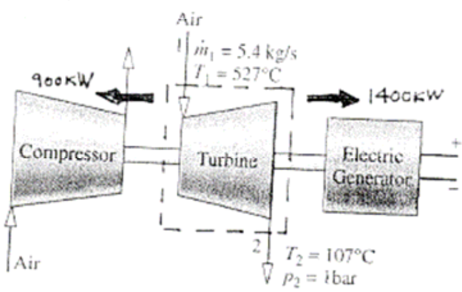


Fig. P4.94

ENGR. MODEL:

1. As shown in the sketch, a control volume encloses the turbine.
2. The control volume is at steady state.
3. The air can be modeled as an ideal gas, and kinetic and potential energy changes are negligible.

ANALYSIS: (a) At steady state, $\dot{m}_1 = \dot{m}_2$. Then, with $\dot{m}_2 = (AV)_2/v_2$,

$$(AV)_2 = \dot{m}_2 v_2 = \dot{m}_2\left[\frac{RT_2}{p_2}\right] = \left(5.4\frac{kg}{s}\right)\left[\frac{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(380\,K)}{10^5\,N/m^2}\right] = 5.89\frac{m^3}{s}$$

(b) An energy rate balance reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

where $\dot{W}_{cv} = 900\,kW + 1400\,kW = 2300\,kW$. With h_1 and h_2 from Table A-22,

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1)$$

$$= 2300\,kW + 5.4\frac{kg}{s}\left(380.77 - 821.95\right)\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= -82\,kW$$

PROBLEM 4.95

KNOWN: Steady-state operating data are provided for a throttling valve in series with a heat exchanger.

FIND: Determine the pressure at the valve exit and the mass flow rate of the liquid stream.

SCHEMATIC & GIVEN DATA:

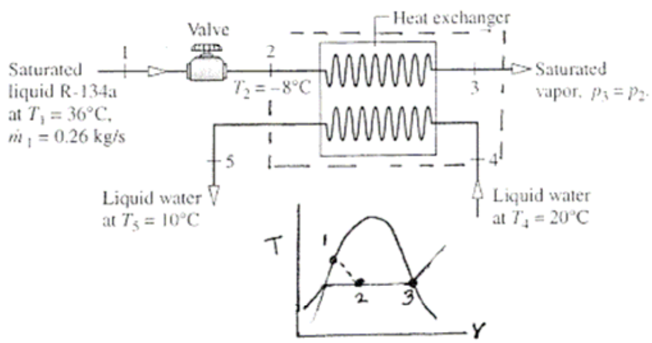


Fig. P4.95

ENGR. MODEL

1. As shown in the sketch, a control volume encloses the heat exchanger.
2. The control volume is at steady state.
3. For the control volume, stray heat transfer and kinetic and potential energy effects can be ignored.
4. The expansion through the valve is a throttling process: $h_2 \approx h_1$. $P_3 \approx P_2$ and $P_5 \approx P_4$.

ANALYSIS:

(a) Since the expansion through the valve is a throttling process, $h_2 \approx h_1$. From Table A-10, $h_1 = 100.25$ kJ/kg. Then, inspection of Table A-10 at $-8°C$ with $h = 100.25$ kJ/kg shows state 2 is in the two-phase region. The pressure is the saturation pressure at $-8°C$: $P_2 = 2.1704$ bar $= 217.04$ kPa. ← P_2

(b) Mass and energy rate balances for the control volume read

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_2 [h_2 - h_3] + \dot{m}_4 [h_4 - h_5]$$

$$\Rightarrow \quad \dot{m}_4 = \frac{\overbrace{\dot{m}_2}^{=\dot{m}_1 = 0.26\ kg/s} [h_3 - h_2]}{[h_4 - h_5]}$$

Since $P_3 = P_2$, $h_3 = h_g(-8°C) = 242.54$ kJ/kg. (Table A-10). For liquid water, $h_4 \approx h_f(T_4)$, $h_5 \approx h_f(T_5)$. Then, with data from Table A-2, $h_4 = 83.96$ kJ/kg and $h_5 = 42.01$ kJ/kg.

$$\therefore \quad \dot{m}_4 = (0.26\ kg/s) \frac{[242.54 - 100.25]}{[83.96 - 42.01]}$$

$$= 0.88\ kg/s$$ ← $\dot{m}_4$

PROBLEM 4.96

KNOWN: A steam turbine at steady state is operated at part load by throttling the steam to lower pressure before it enters the turbine.

FIND: For the turbine, determine the inlet temperature and the power developed per lb of steam flowing.

SCHEMATIC & GIVEN DATA:

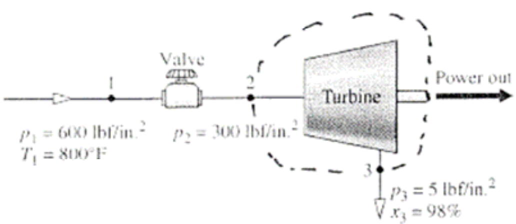


Fig. P4.96

ENGR. MODEL:

1. The control volume enclosing the turbine shown in the sketch is at steady state.
2. For the control volume, $\dot{Q}_{cv}$ and all kinetic and potential energy effects can be ignored.
3. The expansion across the valve is a throttling process: $h_2 = h_1$.

ANALYSIS: (a) Since $h_2 = h_1$, state 2 is fixed by $P_2 = 300$ lbf/in^2 and $h_2 = h_1 = 1407.6$ Btu/lb (from Table A-4E). Interpolating in Table A-4E then gives $T_2 = 774.6$ °F. ← T_2

(b) Energy rate balance:

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[h_2 - h_3 + \cancel{\frac{V_2^2 - V_3^2}{2}} + \cancel{g(z_2 - z_3)}\right]$$

$$\Rightarrow \dot{W}_{cv}/\dot{m} = h_2 - h_3$$

With data from Table A-3E,

$$h_3 = h_f + x_3(h_g - h_f) = 130.17 + 0.98(1000.9) = 1111.05 \text{ Btu/lb}$$

Finally

$$\dot{W}_{cv}/\dot{m} = (1407.6 - 1111.05) \text{ Btu/lb}$$

$$= 296.6 \text{ Btu/lb}$$ ← $\dot{W}_{cv}/\dot{m}$

PROBLEM 4.97

KNOWN: Steady-state operating data are provided for a compressor in series with a condenser.

FIND: Determine the mass flow rate of the Refrigerant 22 and the compressor power.

SCHEMATIC & GIVEN DATA:

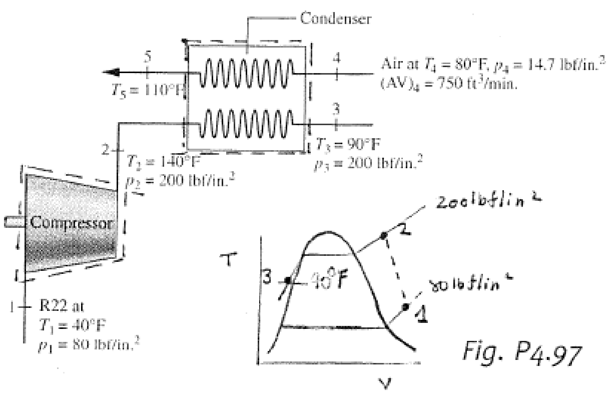


Fig. P4.97

ENGR. MODEL

1. As shown in the sketch, two control volumes are under consideration.
2. Each control volume operates at steady state.
3. For each control volume stray heat transfer and kinetic and potential energy effects can be ignored. For the condenser, $\dot{W}_{cv} = 0$.
4. The air is modeled as an ideal gas.

ANALYSIS:

(a) Mass and energy balances for the control volume enclosing the condenser read

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_R[h_2 - h_3] + \dot{m}_A[h_4 - h_5]$$

$$\Rightarrow \quad \dot{m}_R = \dot{m}_A \left[\frac{h_5 - h_4}{h_2 - h_3}\right]$$

where

$$\dot{m}_A = \frac{[AV]_4}{\underbrace{v_4}_{= \frac{RT_4}{p_4}}} = \frac{p_4 [AV]_4}{R T_4} = \frac{(750\ \text{ft}^3/\text{min})(14.7 \times 144\ \text{lbf/ft}^2)}{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(540^\circ\text{R})} = 55.13\ \frac{\text{lb}}{\text{min}}$$

Then, with $h_2 = 121.25$ Btu/lb from Table A-9E and $h_3 \approx h_f(90°F) = 36.32$ Btu/lb from Table A-7E, together with $h_4 = 129.06$ Btu/lb and $h_5 = 136.26$ Btu/lb from Table A-22E, we get

$$\dot{m}_R = 55.13\ \frac{\text{lb}}{\text{min}} \left[\frac{136.26 - 129.06}{121.25 - 36.32}\right] = 4.67\ \frac{\text{lb}}{\text{min}} \longleftarrow \dot{m}_R$$

(b) Mass and energy rate balances for the control volume enclosing the compressor reduce to

$$\dot{W}_{cv} = \dot{m}_R[\overbrace{h_1}^{= 108.42\ \text{Btu/lb (Table A-9E)}} - h_2]$$

$$= 4.67\ \frac{\text{lb}}{\text{min}} \left(108.42 - 121.25\right) \frac{\text{Btu}}{\text{lb}} \left| \frac{60\ \text{min}}{1\ \text{h}} \right| \left| \frac{1\ \text{hp}}{2545\ \text{Btu/h}} \right|$$

$$= -1.41\ \text{hp} \longleftarrow \dot{W}_{cv}$$

PROBLEM 4.98

4.98 Fig. P4.98 shows part of a refrigeration system consisting of a heat exchanger, an evaporator, a throttling valve, and associated piping. Data for steady-state operation with Refrigerant 134a are given in the figure. There is no significant heat transfer to or from the heat exchanger, valve, and piping. Kinetic and potential energy effects are negligible. Determine the rate of heat transfer between the evaporator and its surroundings, in Btu/h.

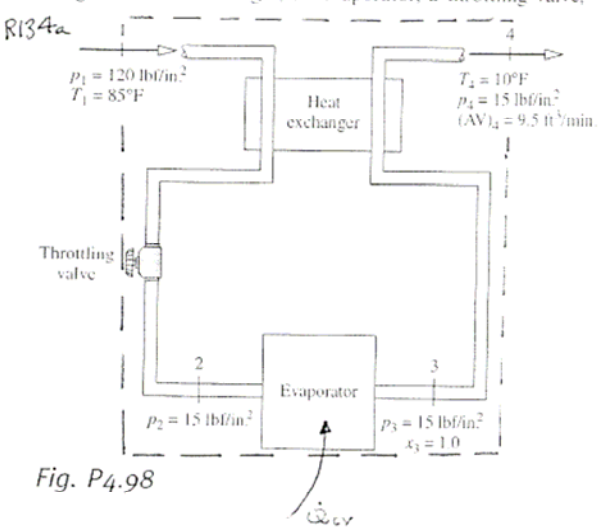


Fig. P4.98

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects are negligible.
3. There is no significant heat transfer to or from the heat exchanger, valve, and piping.

ANALYSIS: Reducing an energy rate balance,

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_4 + \cancel{\frac{V_1^2 - V_4^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m}(h_4 - h_1) \qquad (1)$$

In light of assumption 3, $\dot{Q}_{cv}$ can only account for heat transfer between the evaporator and surroundings.

The required mass flow rate is

$$\dot{m} = \frac{(AV)_4}{v_4} = \frac{9.5\ \text{ft}^3/\text{min}}{3.168\ \text{ft}^3/\text{lb}}\left|\frac{60\ \text{min}}{1\ \text{h}}\right| = 179.92\ \frac{\text{lb}}{\text{h}}$$

where v_4 is from Table A-12E. Also from Table A-12E, $h_4 = 104.38$ Btu/lb.

Referring to Table A-11E, at 120 lbf/in² the saturation temperature is 90.54 °F. Accordingly, at state 1 R134a is a liquid. Using Table A-10E,

$h_1 \approx h_f(T_1) = 38.99$ Btu/lb.

Substituting values into Eq. (1)

$$\dot{Q}_{cv} = \left(179.92\ \frac{\text{lb}}{\text{h}}\right)(104.38 - 38.99)\ \frac{\text{Btu}}{\text{lb}}$$

(1) $$= 11,765\ \text{Btu/h}$$

---

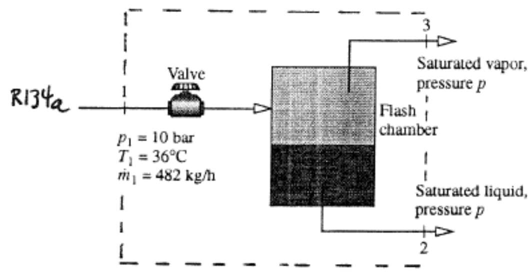
1. Refrigerant flowing through the evaporator of a refrigeration system receives energy by heat transfer from the refrigerated space. See Chap. 10 for discussion.

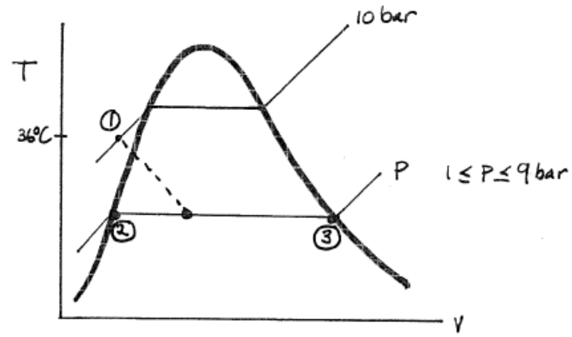
PROBLEM 4.99

KNOWN: Data are provided for a flash chamber operating at steady state. Saturated vapor and saturated liquid streams exit at pressure p.

FIND: If p = 4 bar, determine the mass flow rates of the exiting streams. Plot the mass flow rates of the exiting streams versus pressure p, $1 \leq p \leq 9$ bar.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: 1. The control volume shown in the accompanying figure is at steady state. 2. For the control volume, $\dot{W}_{cv} = 0$, heat transfer with the surroundings is negligible, and kinetic/potential energy effects can be ignored. 3. For the liquid entering at 1, $h_1 \approx h_f(T_1)$.

ANALYSIS: For the control volume, the mass rate balance at steady state gives

$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$, or $\dot{m}_3 = \dot{m}_1 - \dot{m}_2$.

An energy rate balance reduces to

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right] - \dot{m}_2\left[h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{gz_2}\right] - \dot{m}_3\left[h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right]$$

$$\Rightarrow 0 = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 \quad \text{or} \quad 0 = \dot{m}_1 h_1 - \dot{m}_2 h_2 - (\dot{m}_1 - \dot{m}_2) h_3$$

$$\Rightarrow \quad \dot{m}_2 = \dot{m}_1 \left[\frac{h_1 - h_3}{h_2 - h_3}\right]$$

(a) From Table A-10; $h_1 = h_f(36°C) = 100.25$ kJ/kg. Also, at p = 4 bar, Table A-11 gives; $h_2 = h_f(4 \text{ bar}) = 62$ kJ/kg and $h_3 = h_g(4 \text{ bar}) = 252.32$ kJ/kg. Thus

$$\dot{m}_2 = \left(482 \frac{kg}{h}\right)\left[\frac{(100.25) - (252.32)}{(62) - (252.32)}\right] = 385.1 \frac{kg}{h} \longleftarrow \dot{m}_2$$

From above

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 482 - 385.1 = 96.9 \text{ kg/h} \longleftarrow \dot{m}_3$$

(b) The following IT code can be used to generate data for plots of $\dot{m}_2$ and $\dot{m}_3$ versus p:

IT Code

```
p1 = 10  // bar
T1 = 36  // °C
mdot1 = 482  // kg/h
p = 4  // bar
x2 = 0
x3 = 1
mdot1 = mdot2 + mdot3
0 = mdot1 * h1 - mdot2 * h2 - mdot3 * h3
h1 = h_PT("R134A", p1, T1)
h2 = hsat_Px("R134A", p, x2)
h3 = hsat_Px("R134A", p, x3)
```

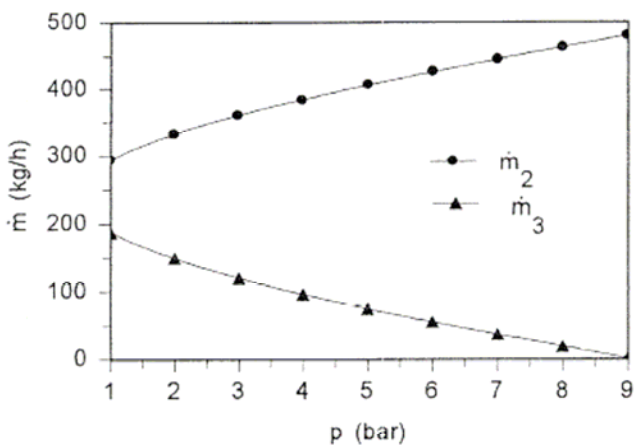
Continued on next slide

Problem 4-99 continued

Using $p = 4$ bar as a sample case

<u>*IT* Results (p = 4 bar)</u>
$\dot{m}_2 = 385.1$ kg/h
$\dot{m}_3 = 96.87$ kg/h
$h_1 = 100.3$ kJ/kg
$h_2 = 62$ kJ/kg
$h_3 = 252.3$ kJ/kg

These results compare very favorable with those of part(a). Now, using the Explore button, sweep p from 1 to 9 bar in steps of 0.1 bar. Then, the following plot can be constructed:

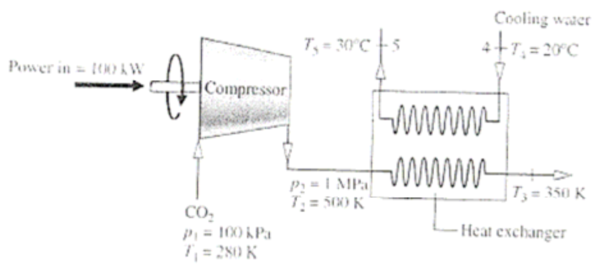


These plots imply that as the pressure in the flash chamber, p, is decreased a greater fraction of the incoming liquid flow "flashes" to saturated vapor.

PROBLEM 4.100

Carbon dioxide (CO_2) modeled as an ideal gas flows through the compressor and heat exchanger shown in Fig. P4.100. The power input to the compressor is 100 kW. A separate liquid cooling water stream flows through the heat exchanger. All data are for operation at steady state. Stray heat transfer with the surroundings can be neglected, as can all kinetic and potential energy changes. Determine (a) the mass flow rate of the CO_2, in kg/s, and (b) the mass flow rate of the cooling water, in kg/s.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. Control volumes at steady state enclose the compressor and heat exchanger.
2. Stray heat transfer and kinetic and potential energy effects are neglected. For the heat exchanger, $\dot{W}_{cv} = 0$.
3. CO_2 is modeled as an ideal gas.

ANALYSIS:

(a) An energy rate balance for the compressor reduces to read

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_1\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{m}_1 = -\frac{\dot{W}_{cv}}{h_2 - h_1} \qquad (1)$$

With enthalpy data on a molar basis from Table A-23

$$h_2 - h_1 = \frac{\bar{h}_2 - \bar{h}_1}{M} = \left(\frac{17{,}678 - 8697}{44.01}\right)\frac{kJ}{kg} = 204.1 \text{ kJ/kg}$$

Then, Eq. (1) gives

$$\dot{m}_1 = \frac{-(-100 \text{ kW})}{204.1 \text{ kJ/kg}}\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right| = 0.49 \text{ kg/s} \longleftarrow \text{(a)}$$

(b) An energy rate balance for the heat exchanger reduces to read

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \underbrace{\dot{m}_2}_{=\dot{m}_1}[h_2 - h_3] + \dot{m}_4[h_4 - h_5]$$

$$\Rightarrow \quad \dot{m}_4 = \left(\frac{h_2 - h_3}{h_5 - h_4}\right)\dot{m}_1 \qquad (2)$$

where $h_4 \approx h_f(T_4) = 83.96$ kJ/kg, $h_5 \approx h_f(T_5) = 125.79$ kJ/kg from Table A-2. Also, from Table A-23

$$h_2 - h_3 = \frac{\bar{h}_2 - \bar{h}_3}{M} = \frac{17678 - 11{,}351}{44.01} = 143.8 \text{ kJ/kg}$$

Then Eq. (2) gives

$$\dot{m}_4 = \left(\frac{143.8 \text{ kJ/kg}}{41.83 \text{ kJ/kg}}\right)\left(0.49\frac{kg}{s}\right) = 1.68\frac{kg}{s} \longleftarrow \text{(b)}$$

PROBLEM 4.101

KNOWN: Steady-state operating data are provided for a waste heat recovery-steam generator - turbine system.

FIND: On the basis of an appropriate analysis, indicate whether, or not, the system should be implemented.

SCHEMATIC & GIVEN DATA:

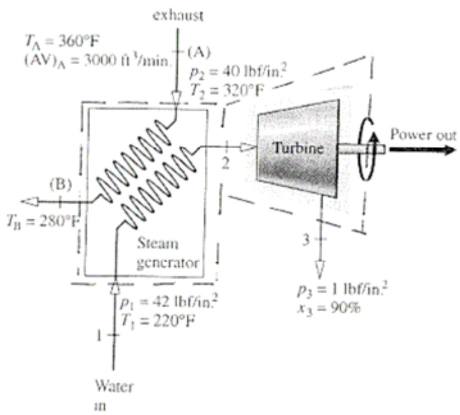


Fig. P4.101

ENGR. MODEL: (1) Both control volumes are at steady state. (2) Heat transfer is negligible between each control volume and its surroundings, and $\dot{W}_{cv}=0$ for the heat exchanger. (3) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas, at a pressure of 14.7 lbf/in². (5) Power is evaluated at 8 cents per kW·h.

ANALYSIS: The turbine power can be determined from an energy balance once the steam flow rate is determined. Begin with a steady-state energy balance on the steam generator

$$0 = \dot{m}_A - \dot{m}_B \Rightarrow \dot{m}_A = \dot{m}_B \equiv \dot{m}_{gas}$$

$$0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}_3 \equiv \dot{m}_{st}$$

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}_{gas}\left[(h_A-h_B) + \cancel{\left(\frac{V_A^2-V_B^2}{2}\right)}^{0} + g\cancel{(z_A-z_B)}^{0}\right] + \dot{m}_{st}\left[(h_1-h_2) + \cancel{\left(\frac{V_1^2-V_2^2}{2}\right)}^{0} + g\cancel{(z_1-z_2)}^{0}\right]$$

$$\dot{m}_{st} = \dot{m}_{gas}\frac{(h_A-h_B)}{(h_2-h_1)}$$

The gas flow rate is

$$\dot{m}_{gas} = \frac{(AV)_A}{v_A} = \frac{p_A(AV)_A}{RT_A}$$

$$= \frac{(14.7\ \text{lbf/in}^2)(3000\ \text{ft}^3/\text{min})}{\left(\frac{1545}{28.97}\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(820\ °\text{R})}\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right| = 145.2\ \text{lb/min}$$

From Table A-22E; $h_A = 196.69$ Btu/lb and $h_B = 177.23$ Btu/lb. For the water, from Table A-2E; $P_1 > P_{sat\,@\,220°F} \Rightarrow$ compressed liquid. Thus, $h_1 \approx h_{f\,@\,220°F} = 188.2$ Btu/lb. At 2, using Table A-4E; $h_2 = 1196.8$ Btu/lb. The flow rate is

$$\dot{m}_{st} = (145.2\ \text{lb/min})\frac{(196.69-177.23)}{(1196.8-188.2)} = 2.8\ \text{lb/min}$$

Continued on next slide

Problem 4-101 continued

Turning now to the control volume enclosing the turbine

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_t + \dot{m}_{st}\left[(h_2 - h_3) + \cancel{\left(\frac{V_2^2 - V_3^2}{2}\right)}^{0} + \cancel{g(z_2 - z_3)}^{0}\right]$$

$$\dot{W}_t = \dot{m}_{st}(h_2 - h_3)$$

Using data from Table A-3E

$$h_3 = h_f + x_3(h_{fg}) = 69.7 + 0.9(1036) = 1002.1 \text{ Btu/lb}$$

Thus

$$\dot{W}_t = \left(2.8 \frac{\text{lb}}{\text{min}}\right)(1196.8 - 1002.1)\frac{\text{Btu}}{\text{lb}}\left|\frac{60 \text{ min}}{1 \text{ h}}\right|\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

$$= 12.85 \text{ hp}$$

Evaluating power at 8 cents per kW·h, the value for $24 \times 365 = 8760$ hours of operation annually is

$$\dot{\$} = (12.85 \text{ hp})\left|\frac{0.7457 \text{ kW}}{1 \text{ hp}}\right|\left(8760 \frac{\text{h}}{\text{year}}\right)\left(\$ \frac{0.08}{\text{kW}\cdot\text{h}}\right)$$

$$= \$\,6715/\text{year}$$

It is unlikely that this value would cover the capital and operating costs associated with the overall system. Accordingly, even though the oven exhaust has a thermodynamic potential for use, the potential is not likely to be exploited by the proposed means owing to economic considerations.

PROBLEM 4.102

KNOWN: Steady-state operating data are provided for a simple steam power plant.

FIND: Determine the thermal efficiency and the mass flow rate of the condenser cooling water.

SCHEMATIC & GIVEN DATA:

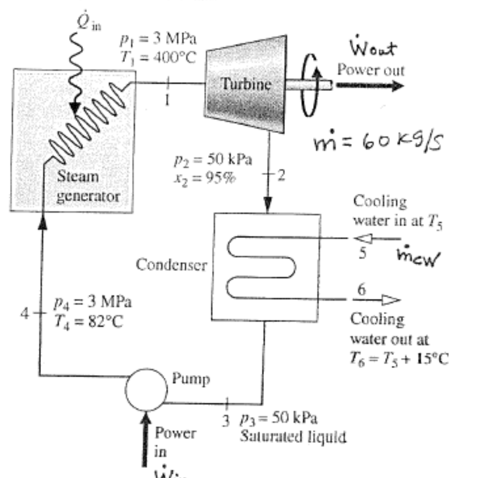


Fig. P4.102

ENGR. MODEL:

1. Control volumes at steady state enclose each of the four components.
2. For each control volume, stray heat transfer and kinetic and potential energy effects are negligible.
3. Energy transfers are in the direction of the arrows.
4. Model the cooling water as incompressible with $(p_6 - p_5) \approx 0$.

ANALYSIS: (a) For any power cycle, the thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}}$$

where $\dot{W}_{cycle} = \dot{W}_{out} - \dot{W}_{in}$. In this case, mass and energy rate balances for control volumes enclosing the turbine and pump reduce to read

$$\dot{W}_{out} = \dot{m}\,[h_1 - h_2], \quad \dot{W}_{in} = \dot{m}\,(h_4 - h_3)$$

where $h_1 = 3230.9\ \frac{kJ}{kg}$ (from Table A-4), $h_2 = h_f + x_2(h_g - h_f) = 340.49 + 0.95(2305.4) = 2530.6$ kJ/kg and $h_3 = 340.9$ kJ/kg (Data from Table A-3), $h_4 = 345.66$ kJ/kg (Table A-5). With these values,

$$\dot{W}_{out} = \dot{m}\,(h_1 - h_2) = 60\,\text{kg/s}\,(3230.9 - 2530.6)\frac{kJ}{kg} = 42{,}018\ \text{kJ/s}$$

$$\dot{W}_{in} = \dot{m}\,(h_4 - h_3) = 60\,\text{kg/s}\,(345.66 - 340.49)\frac{kJ}{kg} = 310\ \text{kJ/s}$$

Mass and energy rate balances for a control volume enclosing the steam generator reduce to give $\dot{Q}_{in} = \dot{m}\,(h_1 - h_4) = 60\,\text{kg/s}\,(3230.9 - 345.66)\frac{kJ}{kg} = 173{,}114$ kJ/s. The thermal efficiency is then

$$\eta = \frac{42{,}018 - 310}{173{,}114} = 0.241\ (24.1\%) \longleftarrow \eta$$

(b) Mass and energy rate balances for a control volume enclosing the condenser reduce to give

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\,[h_2 - h_3] + \dot{m}_{cw}\,[h_5 - h_6]$$

$$\Rightarrow \quad \dot{m}_{cw} = \frac{\dot{m}\,[h_2 - h_3]}{[h_6 - h_5]}$$

where, with assumption 4 and Eq. 3.20b, $(h_6 - h_5) \approx c\,\underbrace{(T_6 - T_5)}_{15\,K} + \cancel{v(p_6 - p_5)}^{\approx 0}$. Using $c = 4.2$ kJ/kg·K from Table A-19, we get

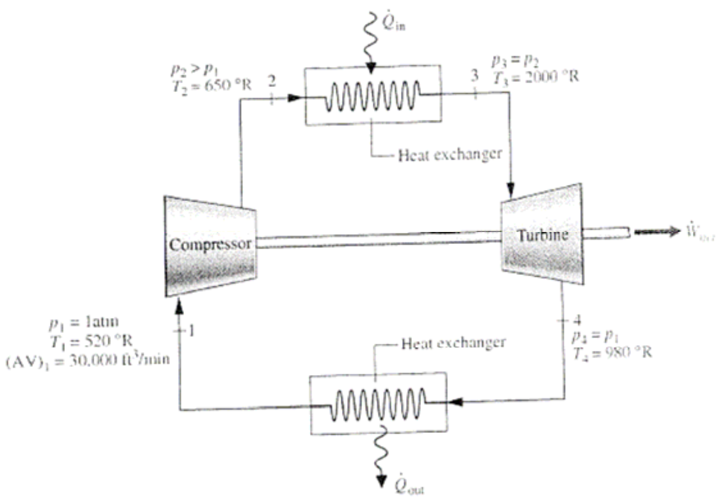
$$\dot{m}_{cw} = 60\,\frac{kg}{s}\left[\frac{2530.6 - 340.49}{(4.2)(15)}\right]$$

$$= 2086\ \text{kg/s} \longleftarrow \dot{m}_{cw}$$

PROBLEM 4.103

A simple gas turbine power cycle operating at steady state with air as the working substance is shown in Fig. P4.103. The cycle components include an air compressor mounted on the same shaft as the turbine. The air is heated in the high-pressure heat exchanger before entering the turbine. The air exiting the turbine is cooled in the low-pressure heat exchanger before returning to the compressor. Kinetic and potential effects are negligible. The compressor and turbine are adiabatic. Using the ideal gas model for air, determine the (a) power required for the compressor, in hp, (b) power output of turbine, in hp, and (c) thermal efficiency of the cycle.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. Control volumes at steady state enclose the compressor, turbine and high-pressure heat exchanger.
2. For the compressor and turbine, $\dot{Q}_{cv} = 0$. For the heat exchanger, $\dot{W}_{cv} = 0$.
3. Kinetic and potential energy effects are negligible.
4. The air is modeled as an ideal gas.

ANALYSIS: Find the mass flow rate as follows:

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1} = \frac{(14.7 \times 144 \frac{lbf}{in^2})(30{,}000\, ft^3/min)}{\left(\frac{1545}{28.97} \frac{ft \cdot lbf}{lb \cdot °R}\right)(520°R)} \left| \frac{60\, min}{1\, h} \right| = 137{,}394 \frac{lb}{h}$$

$(v_1 = RT_1/p_1)$

(a) An energy rate balance for the compressor reduces to read

$$\dot{W}_c = \dot{m}(h_1 - h_2) = \left(137{,}394 \frac{lb}{h}\right)(124.27 - 155.51)\frac{Btu}{lb} \left| \frac{1\, hp}{2545\, Btu/h} \right| = -1687\, hp \quad \text{(a)}$$

where specific enthalpies are from Table A-22E.

(b) An energy rate balance for the turbine reduces to read

$$\dot{W}_t = \dot{m}(h_3 - h_4) = \left(137{,}394 \frac{lb}{h}\right)(504.71 - 236.02)\frac{Btu}{lb} \left| \frac{1\, hp}{2545\, Btu/h} \right| = 14{,}505\, hp \quad \text{(b)}$$

(c)

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_t - |\dot{W}_c|}{\dot{Q}_{in}}$$

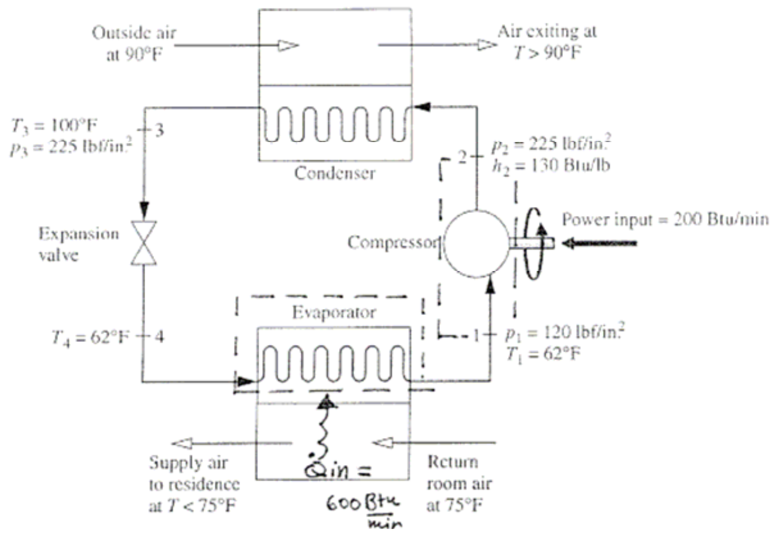
$$= \frac{\dot{m}(h_3 - h_4) - \dot{m}(h_2 - h_1)}{\dot{m}(h_3 - h_2)}$$

$$= \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} = \frac{268.69 - 31.24}{349.20} = 0.68\ (68\%) \quad \text{(c)}$$

PROBLEM 4.104

A residential air conditioning system operates at steady state, as shown in Fig. P4.104. Refrigerant 22 circulates through the components of the system. Property data at key locations are given on the figure. If the evaporator removes energy by heat transfer from the room air at a rate of 600 Btu/min, determine (a) the rate of heat transfer between the compressor and the surroundings, in Btu/min, and (b) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. Control volumes at steady state enclose the compressor and the refrigerant side of the evaporator.
2. Kinetic and potential energy effects can be ignored. For the evaporator, $\dot{W}_{cv} = 0$.
3. The expansion through the valve is a throttling process: $h_4 = h_3$.
4. At state 3, $h_3 \approx h_f(T_3)$

ANALYSIS: (a) To find the mass flow rate of the refrigerant, write an energy rate balance for the control volume enclosing the refrigerant side of the evaporator:

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) \Rightarrow \dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4} \qquad (1)$$

From Table A-9E, $h_1 = 109.88$ Btu/lb (saturated vapor value). Using $h_4 = h_3$, Table A-7E gives $h_3 \approx h_f(T_3) = 39.41$ Btu/lb. Then, Eq. (1) yields

$$\dot{m} = \frac{600 \text{ Btu/min}}{(109.88 - 39.41)} = 8.5 \frac{\text{lb}}{\text{min}}$$

An energy rate balance for the compressor reduces to give

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1) = -200 \frac{\text{Btu}}{\text{min}} + 8.5 \frac{\text{lb}}{\text{min}} (130 - 109.88) \frac{\text{Btu}}{\text{lb}}$$

$$= -29 \text{ Btu/min} \qquad \text{(a)}$$

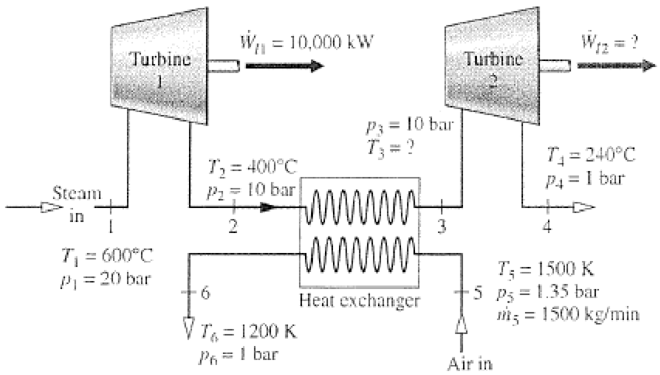
(b) The coefficient of performance is given by (see Sec. 2.6.3)

$$\beta = \frac{\dot{Q}_{in}}{|\dot{W}_{comp}|} = \frac{600 \text{ Btu/min}}{200 \text{ Btu/min}} = 3.0 \qquad \text{(b)}$$

PROBLEM 4.105

Separate streams of steam and air flow through the turbine and heat exchanger arrangement shown in Fig. P4.105. Steady-state operating data are provided on the figure. Heat transfer with the surroundings can be neglected, as can all kinetic and potential energy effects. Determine (a) T_3, in K, and (b) the power output of the second turbine, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. Consider a control volume at steady state enclosing each of the three components.
2. For each control volume, ignore stray heat transfer and kinetic and potential energy effects.
3. The ideal gas model applies for the air. (This can be verified from the compressibility chart.)

ANALYSIS:

(a) To determine the steam mass flow rate write an energy rate balance for turbine 1 and use data from Table A-4:

$$\dot{W}_{t1} = \dot{m}_1 (h_1 - h_2) \Rightarrow \quad \dot{m}_1 = \frac{\dot{W}_{t1}}{h_1 - h_2} = \frac{10{,}000 \text{ kW} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right|}{(3690.1 - 3263.9) \text{ kJ/kg}}$$

$$= 23.46 \text{ kg/s}$$

Next, an energy rate balance for the heat exchanger reduces to

$$0 = \underbrace{\dot{m}_2}_{= \dot{m}_1} [h_2 - h_3] + \dot{m}_5 [h_5 - h_6]$$

with data from Table A-22,

$$\Rightarrow h_3 = h_2 + \frac{\dot{m}_5}{\dot{m}_2} [h_5 - h_6] = 3263.9 \frac{\text{kJ}}{\text{kg}} + \frac{(1500/60) \text{ kg/s}}{23.46 \text{ kg/s}} (1635.97 - 1277.79) \frac{\text{kJ}}{\text{kg}}$$

$$= 3645.6 \frac{\text{kJ}}{\text{kg}}$$

Interpolating in Table A-4 at 10 bar gives, $T_3 = 576$ C ⟵ (a)

(b) An energy rate balance for turbine 2 is

Table A-4

$$\dot{W}_{t1} = \underbrace{\dot{m}_3}_{= \dot{m}_1} (h_3 - h_4) = 23.46 \frac{\text{kg}}{\text{s}} (3645.6 - 2957.5) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

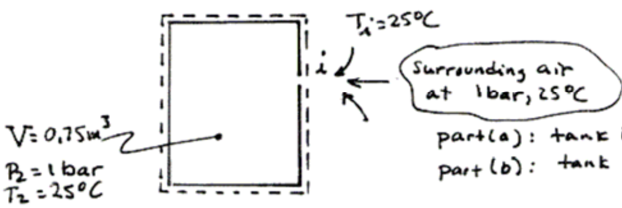
$$= 16{,}213 \text{ kW}$$ ⟵ (b)

PROBLEM 4.106

KNOWN: A pinhole leak develops in the wall of a rigid tank of known volume, and air from the surroundings enters at a known temperature and pressure. The temperature of the air within the tank remains constant at the temperature of the surroundings due to heat transfer from the tank contents.

FIND: Determine the heat transfer between the tank contents and surroundings if (a) the tank is initially evacuated, (b) the tank initially contains air at a specified condition.

SCHEMATIC & GIVEN DATA:



part(a): tank is initially evacuated
part(b): tank initially contains air at 25 °C, 0.7 bar.

ENGR. MODEL: 1. For the control volume shown in the accompanying figure, $\dot{W}_{cv} = 0$, and kinetic/potential energy effects are negligible. 2. The air is modeled as an ideal gas. 3. The condition of the air leaking into the tank at i remains constant at 1 bar, 25°C. 4. The temperature of the air within the tank is 25°C.

ANALYSIS: The mass rate balance for the control volume takes the form $dm_{cv}/dt = \dot{m}_i$. With indicated assumptions, the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + \dot{m}_i h_i \Rightarrow \frac{dU_{cv}}{dt} = \dot{Q}_{cv} + h_i \frac{dm_{cv}}{dt}$$

With assumption 3, h_i remains constant. Thus, on integration

$$\Delta U_{cv} = Q_{cv} + \int_1^2 h_i \, dm_{cv} \Rightarrow m_2 u_2 - m_1 u_1 = Q_{cv} + h_i (m_2 - m_1)$$

Introducing $h_i = u_i + (pv)_i$, and collecting like terms

$$Q_{cv} = \underbrace{m_2(u_2 - u_i)}_{(1)} - \underbrace{m_1(u_1 - u_i)}_{(2)} - (m_2 - m_1)(pv)_i \qquad (1)$$

Since the specific internal energy of an ideal gas depends on temperature only, term (1) vanishes because $T_2 = T_i$. Term (2) vanishes in part (a) because $m_1 = 0$, and in part (b) because $T_1 = T_i$. Accordingly, Eq. (1) reduces in each part to the following working equation:

$$Q_{cv} = -(m_2 - m_1)(pv_i) \qquad (2a)$$

That is, the heat transfer from the tank removes the energy entering the tank at i by flow work (see Sec. 4.4.2). Since $pv_i = RT_i$, Eq. (2a) becomes alternatively

$$Q_{cv} = -(m_2 - m_1) RT_i \qquad (2b)$$

(a) $m_1 = 0$. Using the ideal gas equation of state, $m_2 = p_2 V/RT_2 = p_2 V/RT_i$. So

$$Q_{cv} = -p_2 V = -(1 \text{ bar}) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| (0.75 m^3) \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = -75 \text{ kJ} \longleftarrow$$

(b) $m_1 \neq 0$. Since $m_1 = p_1 V/RT_1 = p_1 V/RT_i$, $m_2 = p_2 V/RT_i$,

$$Q_{cv} = -(p_2 - p_1) V = -(1 \text{ bar} - 0.7 \text{ bar}) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| (0.75 m^3) \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

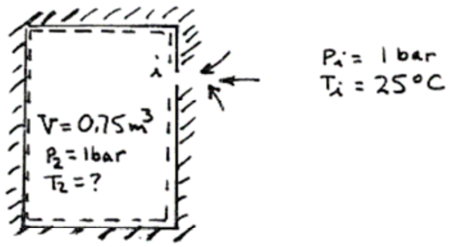
$$= -22.5 \text{ kJ} \longleftarrow$$

PROBLEM 4.107

KNOWN: A pinhole develops in the wall of an initially evaculate tank, and air enters from the surroundings until the pressure is 1 bar.

FIND: Determine the final temperature of the air in the tank in the absence of heat transfer between the tank contents and surroundings.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume is shown in the accompanying figure. 2. For the control volume, $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects are negligible. 3. The air is modeled as an ideal gas. 4. The condition of the air entering the tank remain constant at 1 bar, 21°C.

ANAYSIS: The mass rate balance takes the form $dm_{cv}/dt = \dot{m}_i$. The energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_i h_i$$

Combining the mass and energy rate balance, and integrating using assumption 4

$$\Delta U_{cv} = \int_1^2 \dot{m}_i h_i \, dt \Rightarrow \Delta U_{cv} = h_i \int_1^2 \dot{m}_i \, dt \Rightarrow \Delta U_{cv} = h_i (m_2 - m_1)$$

or since the tank is initially evacuated

$$m_2 u_2 - \cancel{m_1} u_1 = h_i (m_2 - \cancel{m_1}) \Rightarrow u_2 = h_i \qquad (1)$$

From Table A-22 at T_i = 298 K (25°C), h_i = 298.18 kJ/kg. Then,

① interpolating with u_2 = 298.18 kJ/kg, T_2 = 416.5 K (143.5°C) ◄

---

1. To interpret the temperature increase of the air within the tank during the filling process, rewrite Eq. (1) with $h_i = u_i + (pv)_i$ to read

$$u_2 = u_i + \underline{(pv)_i}$$

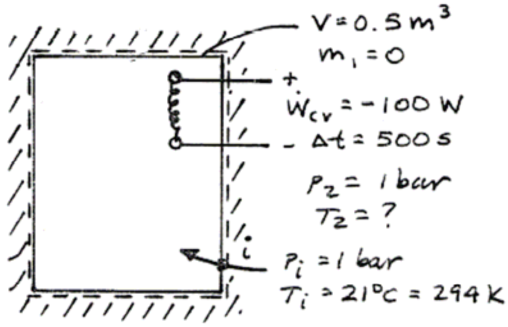
where the underlined term is the flow work discussed in Sec. 4.4.2. Thus, the temperature of air in the tank increases during the process because work is done by the surroundings -- flow work -- on the matter entering the tank.

PROBLEM 4.108

KNOWN: An electric resistor transfers energy to air from the surroundings entering a rigid, well-insulated tank. The final pressure is known.

FIND: Determine the final temperature of air in the tank.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is shown in the accompanying figure. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The air is modeled as an ideal gas. (4) The condition of the air entering the tank remains constant. (5) For the resistor, $\Delta U_{resistor} = 0$.

ANALYSIS; The mass rate balance reduces to $dm_{cv}/dt = \dot{m}_i$, and the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_i h_i$$

Combining and integrating using assumption (4)

$$\Delta U_{cv} = -\int_{t_1}^{t_2} \dot{W}_{cv}\, dt + \int_{t_1}^{t_2} \dot{m}_i h_i\, dt$$

$$= -\dot{W}_{cv}\Delta t + h_i \int_{t_1}^{t_2} \dot{m}_i\, dt = -\dot{W}_{cv}\Delta t + h_i(m_2 - m_1)$$

With $\Delta U_{cv} = m_2 u_2 - m_1 u_1$

$$m_2 u_2 - \cancel{m_1} u_1 = -\dot{W}_{cv}\Delta t + (m_2 - \cancel{m_1})h_i$$

or

$$m_2 u_2 = -\dot{W}_{cv}\Delta t + m_2 h_i \Rightarrow u_2 = \frac{-\dot{W}_{cv}\Delta t}{m_2} + h_i$$

By assumption (3), $m_2 = P_2 V / RT_2$, and

$$u_2 = \left(\frac{RT_2}{P_2 V}\right)(-\dot{W}_{cv}\Delta t) + h_i \qquad (1)$$

Inserting values

$$-\dot{W}_{cv}\Delta t = -(-100\ W)(500\ s)\left|\frac{1\ kJ/s}{10^3\ W}\right| = 50\ kJ$$

$$\frac{RT_2}{P_2 V} = \frac{\left(\frac{8.314}{28.97}\ \frac{kJ}{kg \cdot K}\right)(T_2 \text{ in } K)}{(1\ bar)(0.5\ m^3)}\left|\frac{1\ bar}{10^5\ N/m^2}\right|\left|\frac{10^3\ N \cdot m}{1\ kJ}\right|$$

$$= (5.74 \times 10^{-3})\ T_2$$

From Table A-22; $h_i = 294.17$ kJ/kg. Thus

$$u_2 = \left(5.74 \times 10^{-3}\ \frac{1}{kg \cdot K}\right)(50\ kJ)\ T_2 + 294.17\ kJ/kg \qquad (2)$$

Eq. (2) can be solved for T_2 using an iterative procedure with data for u_2 from Table A-22. The result is

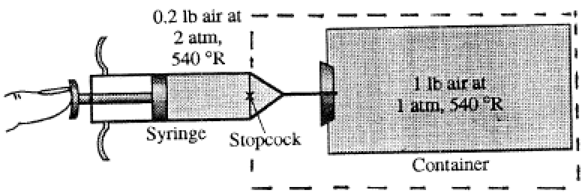
$$T_2 = 665.3\ K = 392.3\ °C$$

PROBLEM 4.109

KNOWN: A small amount of air at a given state is injected into a container initially holding air at a different state.

FIND: Determine the final temperature and pressure of the air in the container.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The control volume is shown in the accompanying schematic. 2. The state of the air remains constant until it passes the stopcock. 3. For the control volume, $\dot{W}_{cv}=0$, heat transfer with the surroundings can be ignored, and kinetic/potential energy effects are negligible. 4. The ideal gas model applies for the air.

ANALYSIS: The mass rate balance reads $dm_{cv}/dt = \dot{m}_i$. The energy rate balance reduces with given assumptions to

① $$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_i h_i \Rightarrow \frac{dU_{cv}}{dt} = h_i \frac{dm_{cv}}{dt}$$

With assumption 2, h_i remains constant. Thus, integration over time gives

$$\Delta U_{cv} = \int_1^2 h_i\, dm_{cv} \Rightarrow \Delta U_{cv} = h_i \Delta m_{cv} \Rightarrow m_2 u_2 - m_1 u_1 = h_i (m_2 - m_1)$$

where 1 and 2 denote the initial and final states within the container, respectively. That is, $m_1 = 1$ lb and $m_2 = 1$ lb $+ 0.2$ lb $= 1.2$ lb. Solving for u_2, and inserting data from Table A-22E for u_1 and h_i

$$u_2 = \frac{m_1 u_1 + (m_2 - m_1) h_i}{m_2} = \frac{(1\text{ lb})[92.04 \frac{\text{Btu}}{\text{lb}}] + (0.2\text{ lb})[129.06 \frac{\text{Btu}}{\text{lb}}]}{1.2\text{ lb}} = 98.21 \frac{\text{Btu}}{\text{lb}}$$

Interpolating in Table A-22E with u_2 gives $T_2 = 576°R$ (116°F).

Using the ideal gas equation of state, $P_2 = m_2 R T_2 / V$, where $V = m_1 R T_1 / P_1$, the final pressure is

$$P_2 = \left(\frac{m_2}{m_1}\right)\left(\frac{T_2}{T_1}\right) P_1 = \left(\frac{1.2\text{ lb}}{1\text{ lb}}\right)\left(\frac{576\,°R}{540\,°R}\right)\left(14.7 \frac{\text{lbf}}{\text{in}^2}\right) = 18.8\text{ lbf/in}^2$$

---

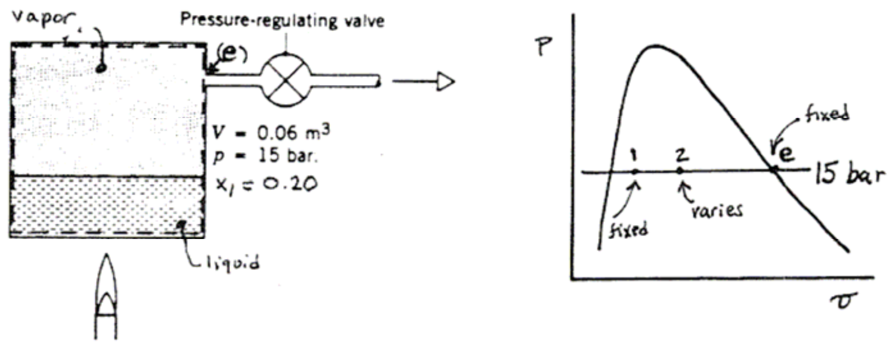
1. The plunger does not directly enter the analysis — it is located in the surroundings of the control volume. However, there is flow work where air enters the control volume. This is incorporated in the $(pv)_i$ term of the specific enthalpy at the inlet, h_i. As shown by the following analysis, the final temperature of the air in the container is higher than the initial temperature. The higher temperature is due to the flow work effect.

PROBLEM 4.110

KNOWN: A rigid tank of known volume initially containing a two-phase liquid-vapor mixture at a known state is heated, allowing saturated vapor to escape while maintaining constant pressure in the tank until a specified final quality is attained.

FIND: (a) If the final quality is 0.5, determine the mass in the tank and the amount of heat transfer. (b) Plot the mass in the tank and the heat transfer versus final quality ranging from 0.2 to 1.0.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume is shown in the schematic. 2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic/potential energy effects are negligible. 3. At exit e the state remains constant, as shown in the p-v diagram.

ANALYSIS: The mass rate balance reads $dm_{cv}/dt = -\dot{m}_e$. With the indicated assumptions, the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e \Rightarrow \frac{dU_{cv}}{dt} = \dot{Q}_{cv} + h_e \frac{dm_{cv}}{dt}$$

Then, with assumption 3, integration over time results in

$$\Delta U_{cv} = Q_{cv} + h_e \int_1^2 dm_{cv} \Rightarrow m_2 u_2 - m_1 u_1 = Q_{cv} + h_e[m_2 - m_1], \text{ or}$$

$$Q_{cv} = m_2 u_2 - m_1 u_1 - h_e[m_2 - m_1] \qquad (1)$$

where $h_e = 2792.2$ kJ/kg, and

$$v_1 = v_f + x_1(v_g - v_f)$$

$$= \left(\frac{1.1539}{10^3}\right) + 0.2\left[0.1318 - \frac{1.1539}{10^3}\right] = 0.02728 \frac{m^3}{kg} \Rightarrow m_1 = \frac{V}{v_1} = \frac{0.06\ m^3}{0.02728\ m^3/kg} = 2.199\ kg$$

$$u_1 = u_f + x_1(u_g - u_f) = 843.16 + 0.2[2594.5 - 843.16] = 1193.4\ kJ/kg$$

With the same values for v_f, v_g, u_f and u_g, $v_2 = v_f + x_2(v_g - v_f)$ and $u_2 = u_f + x_2(u_g - u_f)$. Also, $m_2 = V/v_2$.

(a) $x = 0.5$ Then,

$$v_2 = \left(\frac{1.1539}{10^3}\right) + 0.5\left[0.1318 - \frac{1.1539}{10^3}\right] = 0.06648 \frac{m^3}{kg} \Rightarrow m_2 = \frac{0.06\ m^3}{0.06648\ m^3/kg} = 0.903\ kg. \leftarrow m_2$$

$$u_2 = 843.16 - 0.5[2594.5 - 843.16] = 1718.8\ kJ/kg$$

$$\Rightarrow Q_{cv} = (0.903)(1718.8) - (2.199)(1193.4) - 2792.2(0.903 - 2.199)$$

$$= 2546.5\ kJ \quad \leftarrow Q_{cv}$$

Continued on next slide

Problem 4-110 continued

(b) Data are obtained to make the required plots using *IT*, as follows:

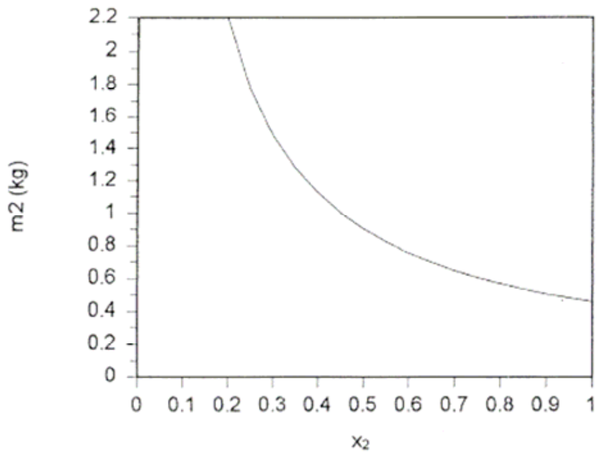
IT Code

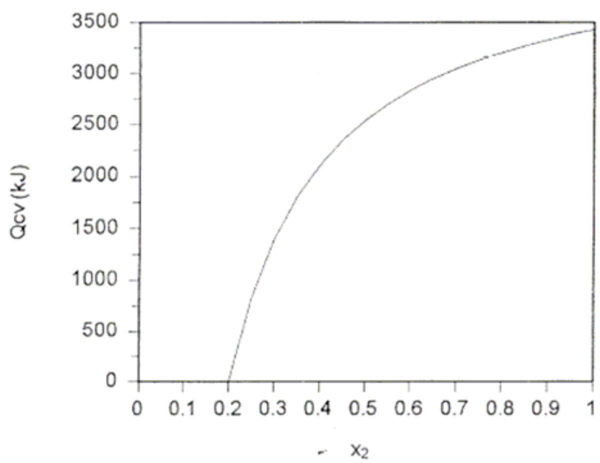
```
V = 0.06  // m³
p = 15  // bar
x1 = 0.2
x2 = 0.5

m1 = V / v1
m2 = V / v2
Qcv = m2 * u2 - m1 * u1 - he * (m2 - m1)

he = hsat_Px("Water/Steam", p, 1)
u1 = usat_Px("Water/Steam", p, x1)
u2 = usat_Px("Water/Steam", p, x2)
v1 = vsat_Px("Water/Steam", p, x1)
v2 = vsat_Px("Water/Steam", p, x2)
```

IT Result ($x_2 = 0.5$)

$Q_{cv} = 2547$ kJ

The result for $x_2 = 0.5$ compares very favorably with the result of part (a). Now, using the Explore button, sweep x_2 from 0.2 to 1.0 in steps of 0.05. The following plots are constructed from the data:





We see from the plots that the heat transfer increases rapidly with x_2 and that the mass in the tank drops rapidly as vapor escapes.

PROBLEM 4.111

As shown in Fig. P4.111, a 300-ft³ tank contains H_2O initially at 30 lbf/in.² and a quality of 80%. The tank is connected to a large steam line carrying steam at 200 lbf/in.², 450°F. Steam flows into the tank through a valve until the tank pressure reaches 100 lbf/in.² and the temperature is 400°F, at which time the valve is closed. Determine the amount of mass, in lb, that enters the tank and the heat transfer between the tank and its surroundings, in Btu.

SCHEMATIC & GIVEN DATA:

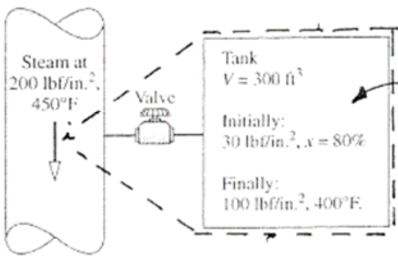


Fig. P4.111

ENGR. MODEL:

1. The control volume is shown on the sketch.
2. Conditions within the steam line remain constant.
3. $\dot{W}_{cv} = 0$. Kinetic and potential energy effects can be neglected.

ANALYSIS: Applying the mass rate balance

$$\frac{dm_{cv}}{dt} = \dot{m}_i \Rightarrow m_2 - m_1 = \int_{t_1}^{t_2} \dot{m}_i \, dt = \left\{ \begin{array}{l} \text{amount of mass that} \\ \text{enters the control volume} \end{array} \right\}$$

In this expression, $m = V/v$. At the initial state, with data from Table A-3E,

$$v_1 = v_f + x v_{fg} = 0.0017 + 0.8(13.75 - 0.0017) = 11.0 \text{ ft}^3/\text{lb}$$

$$\Rightarrow m_1 = \frac{300 \text{ ft}^3}{11.0 \text{ ft}^3/\text{lb}} = 27.27 \text{ lb}$$

At the final state, $v_2 = 4.934 \text{ ft}^3/\text{lb}$ (Table A-4E). So,

$$m_2 = \frac{300 \text{ ft}^3}{4.934 \text{ ft}^3/\text{lb}} = 60.8 \text{ lb}$$

$$\Rightarrow (m_2 - m_1) = (60.8 \text{ lb} - 27.27 \text{ lb}) = 33.53 \text{ lb}$$ ← mass that enters the tank

The energy rate balance reduces as follows:

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}_i \left(h_i + \cancel{\frac{V_i^2}{2}} + \cancel{g z_i}\right) \Rightarrow \frac{dU_{cv}}{dt} = \dot{Q}_{cv} + \dot{m} h_i$$

Integrating and noting that h_i remains constant

$$m_2 u_2 - m_1 u_1 = Q_{cv} - h_i \int_{t_1}^{t_2} \dot{m}_i dt \Rightarrow Q_{cv} = m_2 u_2 - m_1 u_1 - h_i (m_2 - m_1)$$

Using Table A-3E data, $u_1 = u_f + x(u_g - u_f) = 218.84 + 0.8(1088 - 218.84) = 914.17$ Btu/lb.

And from Table A-4E, $u_2 = 1136.2$ Btu/lb, $h_i = 1240.7$ Btu/lb.

Finally

$$Q_{cv} = (60.8 \text{ lb})(1136.2 \text{ Btu/lb}) - (27.27 \text{ lb})(914.17 \text{ Btu/lb}) - 33.53 \text{ lb}(1240.7 \text{ Btu/lb})$$

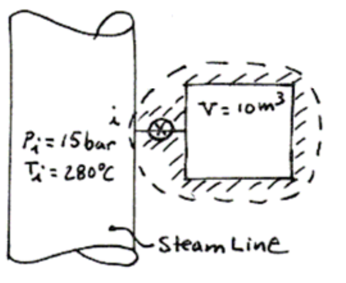
$$= 2551 \text{ Btu}$$ ← Q_{cv}

PROBLEM 4.112

KNOWN: An initially-evacuated, well-insulated tank is filled from a large steam line until the pressure in the tank attains a specified value, p.

FIND: (a) Determine the mass and temperature in the tank when p = 15 bar.
(b) Plot the mass and temperature in the tank versus p ranging from 0.1 to 15 bar.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume is shown in the schematic. 2. For the control volume, $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects are negligible. 3. At inlet i, the state remains constant.

ANALYSIS: The mass rate balance reads $dm_{cv}/dt = \dot{m}_i$. With the indicated assumptions, the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_i h_i \Rightarrow \frac{dU_{cv}}{dt} = h_i \frac{dm_{cv}}{dt}$$

Since h_i is constant (assumption 3), integration over time gives

$$\Delta U_{cv} = h_i \Delta m_{cv} \Rightarrow m_2 u_2 - \cancel{m_1 u_1}^{0} = h_i (m_2 - \cancel{m_1}^{0}) \Rightarrow \qquad u_2 = h_i \qquad (1)$$

(a) p = 15 bar. From Table A-4, h_i = 2992.7 kJ/kg. Then, interpolation at 15 bar with u_2 = 2992.7 kJ/kg ; T_2 = 425°C, v_2 = 0.211 m³/kg. Thus, the final mass is T_2

$$m_2 = \frac{V}{v_2} = \frac{10\,m^3}{0.211\,m^3/kg} = 47.39\,kg \quad m_2$$

(b) IT is used as follows:

IT Code

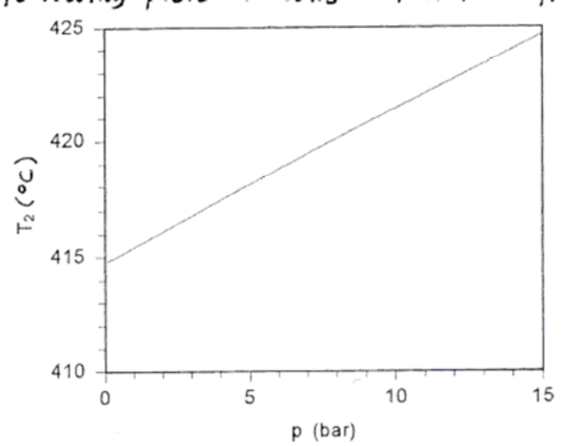
```
V = 10  // m³
pline = 15  // bar
Tline = 280  // °C
p = 15  // bar
u2 = u_PT("Water/Steam", p, T2)
hi = h_PT("Water/Steam", pline, Tline)
u2 = hi
v2 = v_PT("Water/Steam", p, T2)
m2 = V / v2
```

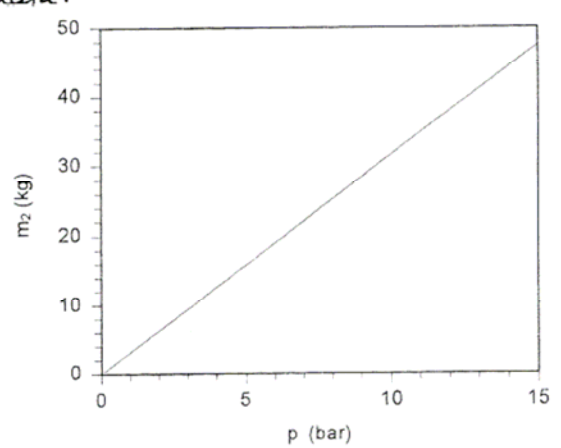
IT Results (p = 15 bar)

T_2 = 424.7 °C
m_2 = 47.39 kg
v_2 = 0.211 m³/kg

These results compare very favorably with the results of part (a).

Now, using the Explore button, sweep p from 0.1 to 15 bar, in steps of 0.1. The following plots are constructed from the data:



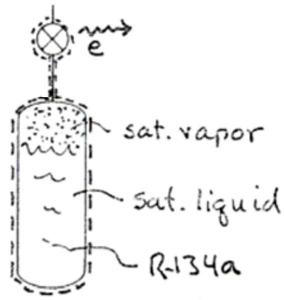


PROBLEM 4.113

KNOWN: A storage tank containing Refrigerant 134a develops a leak allowing saturated vapor to escape until the level of saturated liquid in the tank has dropped to a known value.

FIND: Determine the mass of refrigerant that has escaped and the heat transfer.

SCHEMATIC & GIVEN DATA:



$p = 100\ \text{lbf/in}^2$

$V = 2\ \text{ft}^3$

$V_{f_1} = 1.6\ \text{ft}^3$

$V_{f_2} = 0.8\ \text{ft}^3$

ENGR. MODEL: (1) For the control volume, $\dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The pressure is constant.

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = -\dot{m}_e$. Integrating

$$\int_1^2 \dot{m}_e dt = m_1 - m_2 \quad \text{(mass escaped)}$$

To get the masses, use the given volumes and data from Table A-11E at a pressure of 100 lbf/in²

$$m_{f_1} = \frac{V_{f1}}{v_f} = \frac{(1.6\ \text{ft}^3)}{(0.01332\ \text{ft}^3/\text{lb})} = 120.12\ \text{lb}$$

$$m_{g_1} = \frac{V_{g_1}}{v_g} = \frac{V - V_{f1}}{v_g} = \frac{2 - 1.6}{0.4747} = 0.84\ \text{lb}$$

$$m_1 = m_{f_1} + m_{g_1} = 120.96\ \text{lb}$$

Similarly for state 2

$$m_{f_2} = 60.06\ \text{lb},\ m_{g2} = 2.53\ \text{lb};\ m_2 = 62.59\ \text{lb}$$

and

$$\int_1^2 \dot{m}_e dt = 120.96 - 62.59 = 58.37\ \text{lb} \longleftarrow \text{mass escaped}$$

With assumptions (1) and (2), the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$$

Combining the mass and energy rate balances and integrating

$$Q_{cv} = m_2 u_2 - m_1 u_1 - h_e(m_2 - m_1)$$

from above, $x_1 = m_{g_1}/m_1 = 0.00694$ and $x_2 = m_{g2}/m_2 = 0.04042$. Thus, with data from Table A-11E

$$u_1 = u_f + x_1(u_g - u_f) = 37.21\ \text{Btu/lb};\ u_2 = 39.46\ \text{Btu/lb}$$

and $h_e = h_{g\,@\,100\ \text{lbf/in}^2} = 112.46\ \text{Btu/lb}$

Thus

$$Q_{cv} = (62.59)(39.46) - (120.96)(37.21) - (112.46)(62.59 - 120.96)$$

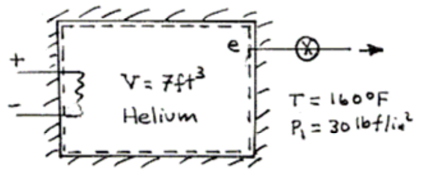
$$= 4533.2\ \text{Btu} \longleftarrow Q_{cv}$$

PROBLEM 4.114

KNOWN: Helium is withdrawn slowly from a well-insulated tank of known volume until the pressure within the tank is p. The temperature within the tank is kept constant by an electrical resistor.

FIND: (a) When p = 18 lbf/in.², determine the mass of helium withdrawn and the energy input to the resistor. (b) Plot the quantities of part (a) versus p ranging from 15 to 30 lbf/in.²

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume is shown in the schematic. 2. For the control volume, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects can be ignored. 3. Helium is modeled as an ideal gas. 4. The mass of the resistor is small enough to be ignored.

ANALYSIS: The mass of helium withdrawn over a time interval equals the difference between the initial amount of mass in the tank and the mass in the tank at the later time: (Amount withdrawn) = $m_1 - m_2$. Since $T_1 = T_2$, the ideal gas equation of state can be used to rewrite this as follows, where $p_2 = p$:

$$\begin{pmatrix}\text{Amount}\\ \text{withdrawn}\end{pmatrix} = \frac{p_1 V}{RT} - \frac{pV}{RT} = (p_1 - p)\frac{V}{RT} \qquad (1)$$

$$= (30-p)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| \frac{(7\ \text{ft}^3)}{\left[\frac{1545}{4.003}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right](620^\circ\text{R})}$$

$$= (30-p)(4.21\times10^{-3})\ \text{lb} \qquad (2)$$

The mass rate balance is $dm_{cv}/dt = -\dot{m}_e$. The energy rate balance reduces to $dU_{cv}/dt = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} - \dot{m}_e h_e$. Introducing the mass rate balance, and noting that h_e remains constant because temperature is constant,

$$\frac{dU_{cv}}{dt} = -\dot{W}_{cv} + h_e\frac{dm_{cv}}{dt} \Rightarrow \Delta U_{cv} = -W_{cv} + h_e\Delta m_{cv} \Rightarrow (-W_{cv}) = m_2u_2 - m_1u_1 - (m_2 - m_1)h_e.$$

Since temperature is constant, $u_2 = u(T)$, $u_1 = u(T)$, $h_e = u(T) + (pv)_e$. Thus

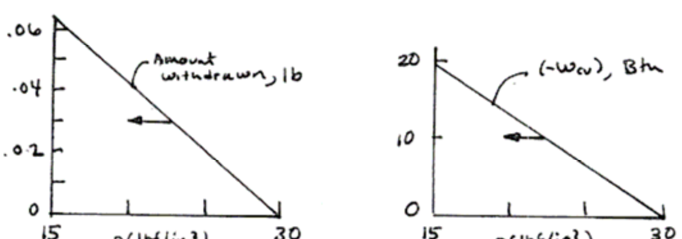
① $(-\dot{W}_{cv}) = (m_1 - m_2)(pv)_e \Rightarrow$ with the ideal gas equation of state, $(-W_{cv}) = (m_1 - m_2)RT$.

By Eq. (1) $(m_1 - m_2) = (p_1 - p)V/RT$, giving $(-W_{cv}) = (p_1 - p)V$. That is,

$$(-W_{cv}) = (30-p)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|(7\ \text{ft}^3)\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = 1.296(30-p)\ \text{Btu} \qquad (3)$$

(a) When p = 18 lbf/in.², Eq. (2) gives (Amt. withdrawn) = 0.0505 kg, Eq. (3) gives $(-W_{cv})$ = 15.55 Btu.

(b) PLOTS



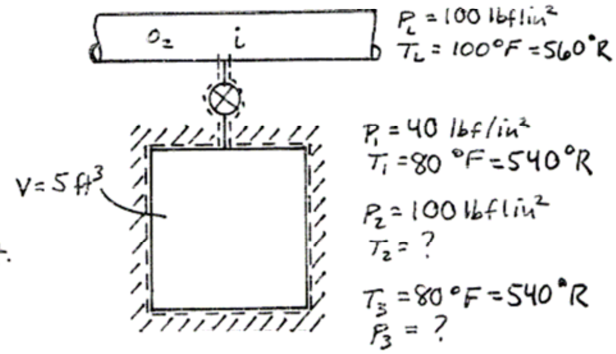
1. To maintain the temperature within the tank constant, the resistor must provide energy to the helium in the tank equal to the energy carried out by flow work at e.

PROBLEM 4.115

KNOWN: An insulated tank containing oxygen (O_2) is connected to a supply line carrying oxygen. A valve between the tank and the line is opened and gas flows into the tank until the pressure reaches that of the line. The valve is closed and eventually the tank contents cool back to their initial temperature.

FIND: Determine (a) the tank temperature when the valve closes and (b) the final pressure in the tank.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) For the control volume shown, $W_{cv} = 0$. (2) There is no heat transfer during the filling process. (3) Kinetic and potential energy effects are negligible. (4) The state of the O_2 in the supply line remains constant. (5) The O_2 behaves as an ideal gas.

ANALYSIS: (a) Consider first the filling process. The mass rate balance takes the form $dm_{cv}/dt = \dot{m}_i$ and with assumptions (1), (2), and (3), the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}_i h_i$$

The specific enthalpy in the supply line is constant. Combining the mass and energy rate balances, and integrating

$$m_2 u_2 - m_1 u_1 = h_i (m_2 - m_1)$$

Using the ideal gas equation of state

$$m_1 = \frac{P_1 V}{R T_1} \quad , \quad m_2 = \frac{P_2 V}{R T_2}$$

Combining and rearranging gives

$$u_2 = h_i + \frac{T_2}{T_1} \cdot \frac{P_1}{P_2} (u_1 - h_i)$$

From Table A-23E

$$h_i = \left(\frac{3886.6 \text{ Btu/lbmol}}{32.00 \text{ lb/lbmol}}\right) = 121.46 \text{ Btu/lb}$$

$$u_1 = 2673.8/32.00 = 83.56 \text{ Btu/lb}$$

Inserting values

$$u_2 = 121.46 - (0.02807) T_2$$

Since u_2 depends on T_2, the final temperature can be found by iteration using data from Table A-23E and this expression. The result is

$$T_2 \approx 661\ °R = 201\ °F \longleftarrow T_2$$

Continued on next slide

Problem 4-115 continued

(b) To determine the pressure after the contents of the tank cool off, first find the mass in the tank after the valve closes

$$m_2 = \frac{P_2 V}{R T_2} = \frac{(100\ \text{lbf/in}^2)(5\ \text{ft}^3)}{\left(\frac{1545}{32.00}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(661\ {}^\circ\text{R})}\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|$$

$$= 2.256\ \text{lb}$$

Thus, the final pressure is

$$P_3 = \frac{m_2 R T_3}{V}$$

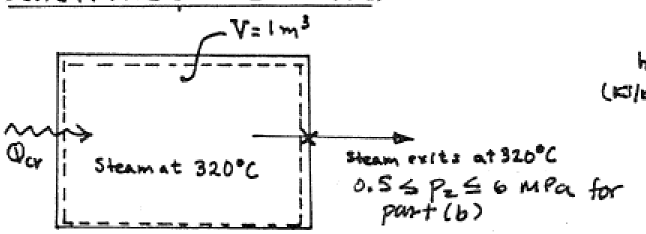
$$= \frac{(2.256)\left(\frac{1545}{32.00}\right)(540)}{(5)\,|144|} = 81.7\ \text{lbf/in}^2 \leftarrow P_2$$

PROBLEM 4.116

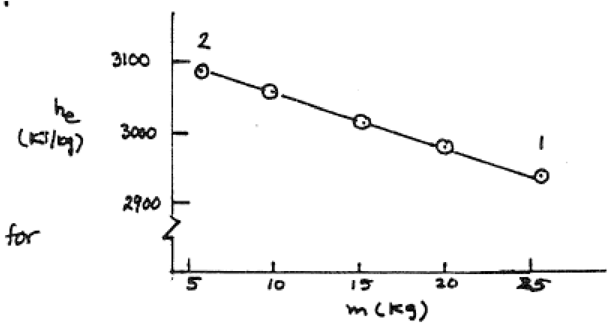
KNOWN: A tank of known volume initially contains steam at a specified state. Steam is withdrawn slowly until the pressure drops to pressure p. The temperature is kept constant by heat transfer to the tank contents.

FIND: (a) Determine the heat tranfer for p = 1.5 MPa. (b) Plot the heat transfer versus p ranging from 0.5 to 6 MPa.

SCHEMATIC & GIVEN DATA:



$p_1 = 6\text{ MPa}$, $p_2 = 1.5\text{ MPa}$ (part a)



ENGR. MODEL: (1) For the control volume shown in the figure, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects can be ignored. (2) At each instant, pressure is uniform throughout the steam.

ANALYSIS: (a) The mass rate balance takes the form $dm_{cv}/dt = -\dot{m}_e$. Using the assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$$

Introducing the mass rate balance and integrating

$$\Delta U_{cv} = Q_{cv} + \int_1^2 h_e dm_{cv} \Rightarrow Q_{cv} = m_2 u_2 - m_1 u_1 - \int_1^2 h_e dm \qquad (1)$$

where m denotes the mass contained within the tank. At any instant, $m = V/v$ where v is specific volume at that instant determined by 320°C and the tank pressure. Initially, $p_1 = 6\text{ MPa}$, so Table A-4 gives $v_1 = 0.03876\text{ m}^3/\text{kg}$, $u_1 = 2720\text{ kJ/kg}$. Finally, $p_2 = 1.5\text{ MPa}$, so $v_2 = 0.1765\text{ m}^3/\text{kg}$, $u_2 = 2817.1\text{ kJ/kg}$. Thus

$$m_1 = \frac{V}{v_1} = \frac{1\text{ m}^3}{0.03876\text{ m}^3/\text{kg}} = 25.8\text{ kg} \quad , \quad m_2 = \frac{V}{v_2} = \frac{1\text{ m}^3}{0.1765\text{ m}^3/\text{kg}} = 5.67\text{ kg} \qquad (2)$$

For each of several pressures in the interval $1.5\text{ MPa} < p < 6.0\text{ MPa}$, the mass within the tank can be calculated in like manner and the specific enthalpy h_e determined from Table A-4 at 320°C and the specified pressure. These values allow the plot of h_e vs m given above to be constructed. The area under the line from 1 to 2 equals the term $-\int_1^2 h_e dm$ of Eq. (1). As this variation is very nearly linear, the average value of h_e can be used to evaluate this term:

$$-\int_1^2 h_e dm = \left(\frac{(h_e)_1 + (h_e)_2}{2}\right)(m_1 - m_2) = \left(\frac{3081.9 + 2952.6}{2}\right)(25.8 - 5.67)$$
$$= \left(3017.25 \frac{\text{kJ}}{\text{kg}}\right)(20.13\text{ kg}) = 60737\text{ kJ}$$

Finally, inserting values into Eq. (1)

$$Q_{cv} = (5.67\text{ kg})\left(2817.1 \frac{\text{kJ}}{\text{kg}}\right) - (25.8\text{ kg})\left(2720 \frac{\text{kJ}}{\text{kg}}\right) + 60737\text{ kJ}$$

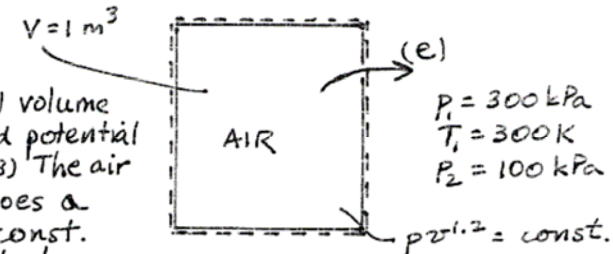
$$= 6534\text{ kJ} \longleftarrow Q_{cv}$$

PROBLEM 4.117

KNOWN: Air escapes slowly from a tank. The initial state and final pressure are known. The that remains in the tank undergoes a polytropic process.

FIND: Determine the heat transfer for a control volume enclosing the tank.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) For the control volume shown, $\dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The air remaining in the tank undergoes a process described by $pv^{1.2} = $ const. (4) The air is modeled as an ideal gas with constant specific heats.

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = -\dot{m}_e$. With the assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e \qquad (1)$$

Since the process occurs slowly, the state of the air in the tank is uniform at any time. Thus, $U_{cv} = mu$, and $h_e = u + RT$. Inserting these expressions into (1) along with the mass rate balance

$$m\frac{du}{dt} + \cancel{u\frac{dm}{dt}} = \dot{Q}_{cv} + (\cancel{u} + RT)\frac{dm}{dt}$$

Thus $$\dot{Q}_{cv}\,dt = m\,du - RT\,dm = m\,c_v\,dT - RT\,dm \qquad (2)$$

Each term on the right side of (2) can be expressed in terms of pressure, as follows. First, from $pv^{1.2} = $ constant

$$m = \frac{V}{v} = \left(\frac{p}{\text{const}}\right)^{\frac{1}{1.2}} V = c\,p^{\frac{1}{1.2}}$$

and $$dm = c\left(\frac{1}{1.2}\right) p^{\left(\frac{1}{1.2}-1\right)} dp \qquad (3)$$

Further $$RT = \frac{pV}{m} = \frac{pV}{c\,p^{1/1.2}} = \frac{V}{c}\,p^{\left(1-\frac{1}{1.2}\right)} \qquad (4)$$

and $$dT = \frac{V}{Rc}\left(1 - \frac{1}{1.2}\right) p^{(-1/1.2)} dp \qquad (5)$$

Substituting (3), (4) and (5) into (2) gives

$$\dot{Q}_{cv}\,dt = \left[c\,p^{\frac{1}{1.2}}\right] c_v \left[\frac{V}{Rc}\left(1 - \frac{1}{1.2}\right) p^{\left(-\frac{1}{1.2}\right)} dp\right] - \left[\frac{V}{c}\,p^{\left(1-\frac{1}{1.2}\right)}\right]\left[c\left(\frac{1}{1.2}\right) p^{\left(\frac{1}{1.2}-1\right)} dp\right]$$

$$= \left[\frac{c_v}{R}\left(1 - \frac{1}{1.2}\right) - \left(\frac{1}{1.2}\right)\right] V\,dp$$

Integrating

$$Q_{cv} = \left[\frac{c_v}{R}\left(1 - \frac{1}{1.2}\right) - \left(\frac{1}{1.2}\right)\right] V\,(p_2 - p_1)$$

Continued on next slide

Problem 4-117 continued

From Table A-20, $c_v \approx 0.717$ kJ/kg·K. Thus

$$Q_{cv} = \left[\frac{(0.717\ \text{kJ/kg}\cdot\text{K})}{\left(\frac{8.314}{28.97}\cdot\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\left(1-\frac{1}{1.2}\right)-\left(\frac{1}{1.2}\right)\right](1\,\text{m}^3)(100-300)\,\text{kPa}\left|\frac{10^3\,\text{N/m}^2}{1\,\text{kPa}}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

① $= 83.39$ kJ $\qquad Q_{cv}$

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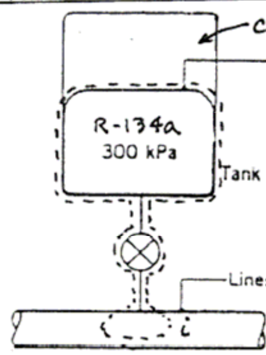
1. The positive sign denotes that the energy transfer is into the control volume.

PROBLEM 4.118

KNOWN: A well-insulated tank containing R-134a is connected to a supply line. As refrigerant is allowed to flow into the tank, a flexible bladder in the tank expands to maintain the refrigerant in the tank at constant pressure.

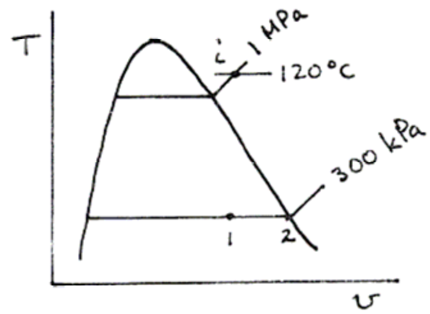
FIND: Determine the amount of mass admitted to the tank between the initial time and the instant when all the liquid in the tank is vaporized.

SCHEMATIC & GIVEN DATA:



Initially $x_1 = 0.8$
$m_1 = 25$ kg

Finally $x_2 = 1.0$



ENGR. MODEL: (1) The control volume is shown with $\dot{Q}_{cv} = 0$. (2) Conditions in the supply line remain constant. (3) The pressure remains constant in the tank. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: The mass rate balance takes the form; $dm_{cv}/dt = \dot{m}_i$. With the assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}_i h_i$$

The specific enthalpy h_i is constant by assumption (2). Thus, combining the mass and energy rate balances and integrating

$$\Delta U_{cv} = -W_{cv} + \int_{t_1}^{t_2} \dot{m}_i h_i \, dt$$

Since h_i is constant

$$\Delta U_{cv} = -W_{cv} + h_i \int_{t_1}^{t_2} \dot{m}_i \, dt = -W_{cv} + h_i (m_2 - m_1) \qquad (1)$$

To evaluate the work, note that the pressure in the tank is constant. Thus

$$W_{cv} = \int p \, dV = p(V_2 - V_1) = p(m_2 v_2 - m_1 v_1) \qquad (2)$$

Combining (1) and (2), and noting that $\Delta U_{cv} = m_2 u_2 - m_1 u_1$

$$m_2 u_2 - m_1 u_1 = -p(m_2 v_2 - m_1 v_1) + h_i (m_2 - m_1)$$

or

$$m_2 [(u_2 + p v_2) - h_i] = m_1 [(u_1 + p v_1) - h_i]$$

$$m_2 [h_2 - h_i] = m_1 [h_1 - h_i]$$

Continued on next slide

Problem 4-118 continued

Solving for m_2

$$m_2 = m_1\left(\frac{h_1 - h_i}{h_2 - h_i}\right)$$

Using data from Table A-11 at 3bar: $h_f = 50.85$, $h_g = 247.59$ kJ/kg

$$h_1 = (50.85) + (0.8)[247.59 - 50.85] = 208.24 \text{ kJ/kg}$$
$$h_2 = 247.59 \text{ kJ/kg}$$

From Table A-12, $h_i = 356.52$ kJ/kg. Thus

$$m_2 = 25\text{kg}\left(\frac{208.24 - 356.52}{247.59 - 356.52}\right) = 34.03 \text{ kg}$$

Finally,

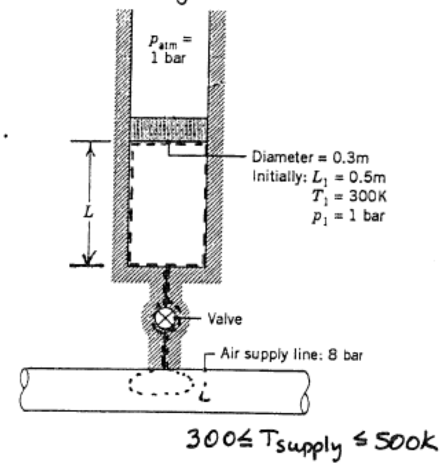
$$\Delta m = m_2 - m_1$$
$$= 34.03 - 25 = 9.03 \text{ kg}$$

PROBLEM 4.119

KNOWN: Air is admitted slowly into an insulated piston-cylinder assembly from a supply line until the volume inside the cylinder has doubled.

FIND: Plot the final temperature and mass inside the cylinder for a given range of supply temperatures.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) For the control volume shown, $\dot{Q}_{cv} = 0$. (2) Kinetic and potential energy effects can be neglected. (3) The weight of the piston and friction between the piston and cylinder wall can be neglected. (4) The air behaves as an ideal gas.

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = \dot{m}_i$. With the assumptions listed, the energy balance is

$$\frac{dU_{cv}}{dt} = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_i h_i$$

The line conditions are constant, so h_i is constant. Combining the mass and energy rate balances and integrating

$$m_2 u_2 - m_1 u_1 = -W_{cv} + h_i (m_2 - m_1) \qquad (1)$$

To evaluate the work, note that the pressure in the cylinder is always atmospheric since the process is slow and since assumption (3) applies. Thus

$$W_{cv} = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1)$$

From the given data

$$V_1 = \left(\frac{\pi d_{pist}^2}{4}\right) L_1 = \left(\frac{\pi (.3^2)\text{m}^2}{4}\right)(0.5\text{ m}) = 0.03534\text{ m}^3$$

and $\quad V_2 = 0.07068\text{ m}^2$

$$W_{cv} = (1\text{ bar})(0.07068 - 0.03534)\text{m}^3 \left|\frac{10^5 \text{N/m}^2}{1\text{bar}}\right| \left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= 3.534\text{ kJ}$$

From Table A-22, at $T_1 = 300\text{K}$; $u_1 = 214.07$ kJ/kg. Also, $h_i = h_i(T_{supply})$.
For $T_{supply} = 300\text{K}$, $h_i = 300.19$ K.

Using the ideal gas model

$$m_1 = \frac{P_1 V_1}{RT_1} = \frac{(1\text{ bar})(0.03534\text{ m}^3)}{\left(\frac{8.314}{28.97}\cdot\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(300\text{K})}\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right| = 0.04105\text{ kg}$$

and
$$m_2 = \frac{P_2 V_2}{RT_2} = \frac{(1)(0.07068)|100|}{\left(\frac{8.314}{28.97}\right)(T_2)} = \frac{24.628}{T_2} \qquad (2)$$

Incorporating these results into (1) and rearranging

Continued on next slide

Problem 4-119 continued

$$m_2 u_2 - h_i (m_2 - m_1) = m_1 u_1 - W_{cv}$$

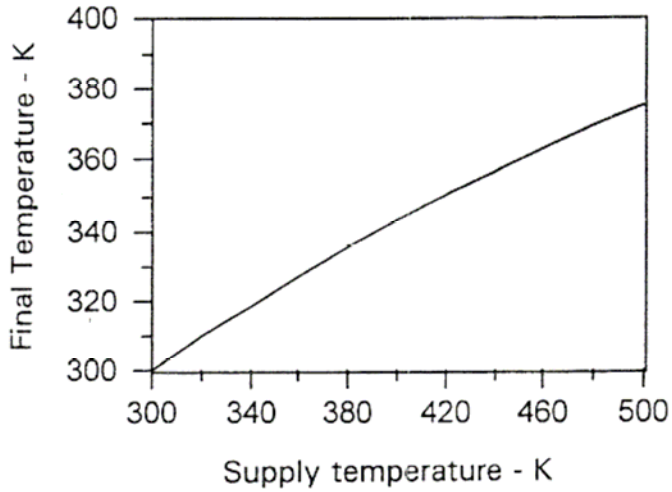
$$\frac{24.628\, u_2}{T_2} - h_i \left(\frac{24.628}{T_2} - 0.04105 \right) = 5.254 \qquad (3)$$

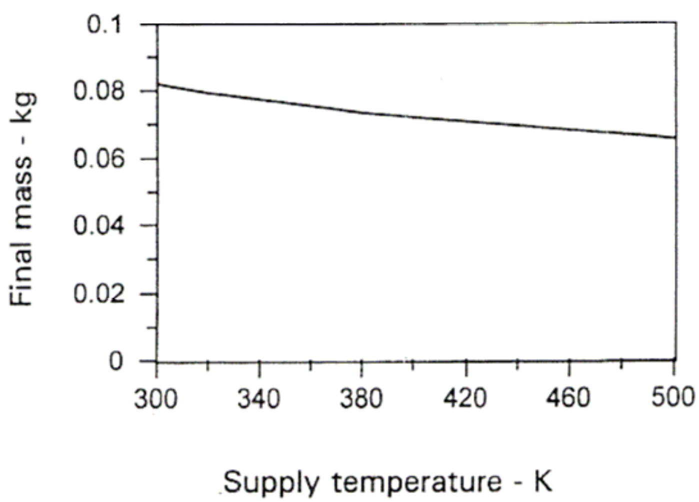
Equations (2) and (3) can be solved for given values of $h_i (T_{supply})$ by an iterative procedure and data from Table A-22. For $T_{supply} = 300K$,

$$T_2 = 300K$$

$$m_2 = 0.0821 \text{ kg}$$

Plotting for the range of T_{supply} values





PROBLEM 4.120

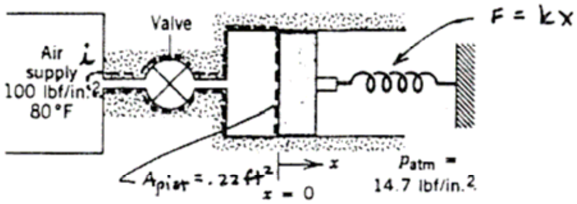
KNOWN: A well-insulated piston-cylinder assembly is connected by a valve to an air supply. The supply conditions and the initial state of air inside the cylinder. Air is admitted slowly causing the piston to compress a spring. The initial and final volumes within the cylinder are known.

FIND: Plot the final pressure and final temperature within the cylinder versus spring constant k varying from 650 to 750 lbf/ft.

SCHEMATIC & GIVEN DATA:

Initially: $p_1 = 14.7\ \text{lbf/in}^2$
$T_1 = 80^\circ F$
$V_1 = 0.1\ ft^3$

Finally: $V_2 = 0.4\ ft^3$



ENGR. MODEL: (1) The control volume is shown on the accompanying diagram, with $\dot{Q}_{cv} = 0$ (2) Conditions in the air supply remain constant. (3) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas. (5) There is no friction between the piston and cylinder wall.

ANALYSIS: The final pressure is found by applying Newton's Law to the piston. Since air is admitted slowly; $\Sigma F_x = 0$. Thus

$$p A_{pist} = p_{atm} A_{pist} + kx$$

with $V - V_1 = x A_{pist}$, the pressure is

$$p = p_{atm} + \frac{k(V - V_1)}{A_{pist}^2} \qquad (a)$$

At $V = V_2$

$$p_2 = 14.7\ \frac{lbf}{in^2} + \frac{(k\ lbf/ft)(0.4 - 0.1)ft^3}{(.22^2)\ ft^4}\left(\frac{1\ ft^2}{144\ in^2}\right)$$

$$= \left[14.7 + 0.043\,k\right] lbf/in^2 \qquad (1)$$

The mass rate balance takes the form; $dm_{cv}/dt = \dot{m}_i$. With assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}_i h_i$$

The specific enthalpy h_i is constant by assumption (3). Thus, combining the mass and energy rate balances and integrating

$$\Delta U_{cv} = -W_{cv} - \int_1^2 h_i\, dm_{cv} \Rightarrow m_2u_2 - m_1u_1 = -W_{cv} + h_i(m_2 - m_1) \qquad (2)$$

Since the process occurs slowly, $W_{cv} = \int p\,dV$. With Eq.(a) above

$$W_{cv} = \int_{V_1}^{V_2}\left[\left(p_{atm} - \frac{kV_1}{A_{pist}^2}\right) + \frac{kV}{A_{pist}^2}\right]dV$$

$$= \left(p_{atm} - \frac{kV_1}{A_{pist}^2}\right)(V_2 - V_1) + \frac{k(V_2^2 - V_1^2)}{2A_{pist}^2}$$

Continued on next slide

Problem 4-120 continued

$$\therefore W_{cv} = p_{atm}(V_2 - V_1) + \frac{k}{(A_{pist})^2}\left[\frac{V_2^2 - V_1^2}{2} - V_1(V_2 - V_1)\right]$$

$$= p_{atm}(V_2 - V_1) + \frac{k}{2(A_{pist})^2}\left[V_2 - V_1\right]^2$$

$$= \left[14.7\ \frac{lbf}{in^2}\left|\frac{144\ in^2}{1\ ft^2}\right|(0.3\ ft^3) + \frac{k\ (lbf/ft)}{2(0.22\ ft^2)^2}\left[0.3\ ft^3\right]^2\right]\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right|$$

$$= \left[0.816 + (1.195\times10^{-3})k\right]\ Btu \qquad (3)$$

Also, with the ideal gas model equation of state

$$m_1 = \frac{p_1 V_1}{R T_1} = \frac{(14.7\times144\ lbf/ft^2)(0.1\ ft^3)}{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(540°R)} = 7.35\times10^{-3}\ lb$$

$$m_2 = \frac{p_2 V_2}{R T_2}, \text{ where } p_2 \text{ is given by Eq. (1)} \qquad (4)$$

Since $T_1 = T_i = 540°R\ (80°F)$, $u_1 = u(540°R)$, $h_i = h(540°R)$.
Accordingly, T_2 can be determined by solving Eq. (2), together with Eqs. (1), (3), (4) and known values of V_2, u_1, and h_i.
This scheme is implemented via the following IT code:

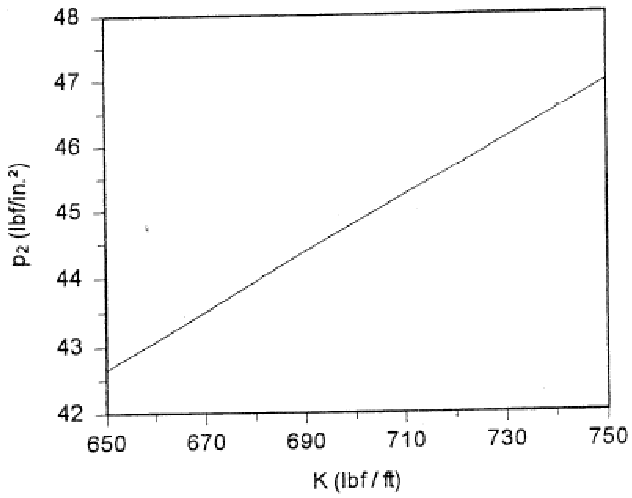
IT Code

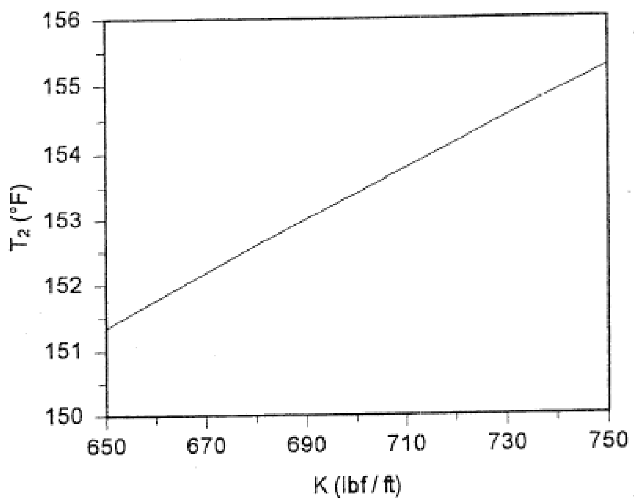
```
p1 = 14.7  // lbf/in.²
T1 = 80  // °F
Ti = T1
V1 = 0.1  // ft²
V2 = 0.4  // ft²
K = 650 // lbf/ft

p2 = 14.7 + 0.043 * K
m2 * u2 - m1 * u1 = - Wcv + hi * (m2 - m1)
Wcv = 0.816 + 1.195E-3 * K
m1 = V1 / v1
m2 = V2 / v2

u1 = u_T("Air", T1)
v1 = v_TP("Air",T1,p1)
u2 = u_T("Air", T2)
v2 = v_TP("Air",T2,p2)
hi = h_T("Air", Ti)
```

Now, using the Explore button, sweep K from 650 to 750 lbf/ft in steps of 1, to get the following plots:

Continued on next slide

Problem 4-120 continued





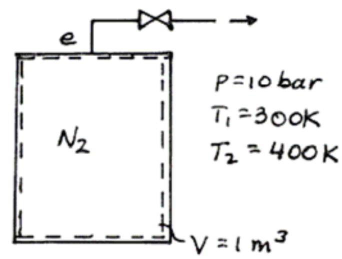
Note that the pressure plot is linear, as indicated by Eq. (1), and could have been readily plotted by hand. However, the temperature plot is non-linear and would be quite difficult to obtain without the use of IT or other software.

PROBLEM 4.121

KNOWN: Heat transfer occurs to nitrogen gas contained in a rigid tank. Gas escapes through a pressure relief valve, maintaining constant pressure in the tank. The initial and final temperatures are specified.

FIND: Determine the mass of nitrogen that escapes and the amount of energy transfer by heat.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) For the control volume shown, $\dot{W}_{cv} = 0$. (2) The state in the control can be assumed to be uniform at any time during the process. (3) Kinetic and potential energy effects can be neglected. (4) The nitrogen behaves as an ideal gas with constant specific heats evaluated at 350K.

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = -\dot{m}_e$. Thus the mass that escapes is

$$\int_1^2 \dot{m}_e\, dt = -\int_{m_1}^{m_2} dm_{cv} = m_1 - m_2$$

With the ideal gas equation of state

$$\int_1^2 \dot{m}_e\, dt = \frac{PV}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$= \frac{(10\text{ bar})(1\text{ m}^3)}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\left[\frac{1}{300\text{K}} - \frac{1}{400\text{K}}\right]\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= 2.904\text{ kg} \longleftarrow \text{mass escaped}$$

With the assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$$

Noting that $U_{cv} = mu$ and $h_e = u + RT$, and with the mass rate balance

$$m\frac{du}{dt} + \cancel{u}\frac{dm}{dt} = \dot{Q}_{cv} + (\cancel{u} + RT)\frac{dm}{dt}$$

$$\dot{Q}_{cv} = m\frac{du}{dt} - RT\frac{dm}{dt}$$

For an ideal gas, $du = c_v\, dT$. Also, with $m = PV/RT$

$$dm = \left(\frac{PV}{R}\right)\frac{dT}{(-T^2)}$$

Thus

$$\dot{Q}_{cv} = \left(\frac{PV}{RT}\right)c_v\frac{dT}{dt} + \left(\frac{PV}{RT}\right)R\frac{dT}{dt} = \frac{PV}{R}(c_v + R)\frac{1}{T}\frac{dT}{dt}$$

Continued on next slide

Problem 4-121 continued

Noting that $c_v + R = c_p$

$$\dot{Q}_{cv}\,dt = \left(\frac{PVc_p}{R}\right)\frac{dT}{T}$$

From Table A-20, $c_p = 1.041$ kJ/kg·K. Integrating and inserting values

$$\int_1^2 \dot{Q}_{cv}\,dt = \left(\frac{PVc_p}{R}\right)\int_{T_1}^{T_2}\frac{dT}{T}$$

$$Q_{cv} = \left(\frac{PVc_p}{R}\right)\ln\frac{T_2}{T_1}$$

$$= \frac{(10\text{ bar})(1\text{ m}^3)(1.041\text{ kJ/kg}\cdot\text{K})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\ln\left(\frac{400}{300}\right)\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

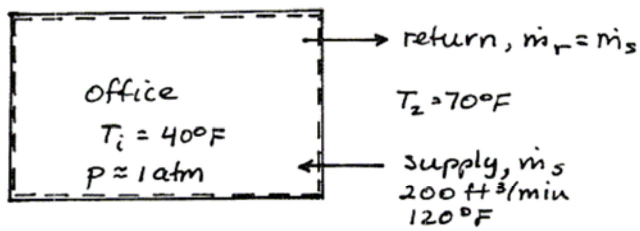
$$= 1043\text{ kJ} \leftarrow Q_{cv}$$

PROBLEM 4.122

KNOWN: The air supply to an office is shut off overnight and the room temperature drops. In the morning, the thermostat is reset and a known volumetric flow rate of heated air is supplied.

FIND: Estimate the time it takes for the room to reach 70°F, and plot room temperature as a function of time.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) For the control volume shown, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The air behaves as an ideal gas with constant specific heats. (3) The pressure is taken as 1 atm everywhere. (4) Kinetic and potential energy effects can be neglected. (5) The room air is well-mixed.

ANALYSIS: The mass rate balance takes the form $dm/dt = \dot{m}_s - \dot{m}_r$. With $\dot{m}_s = \dot{m}_r$; $dm/dt = 0$. Thus, the mass in the room is constant with time. The energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}_s h_s - \dot{m}_r h_r$$

or, with $U_{cv} = mu$ and $h_r = h(T)$

$$m\frac{du}{dt} + u\cancel{\frac{dm}{dt}}^{\,0} = \dot{m}_s\left(h_s - h(T)\right)$$

Further, $du = c_v\,dT$ and $h_s - h(T) = c_p(T_s - T)$. Thus

$$m c_v \frac{dT}{dt} = \dot{m}_s c_p (T_s - T)$$

With $dT = -d(T_s - T)$ and $c_p/c_v = k$

$$-\frac{m\,d(T_s - T)}{k\,(T_s - T)} = \dot{m}_s\,dt \qquad (1)$$

Evaluating $\dot{m}_s$

$$\dot{m}_s = \frac{(AV)_s}{v_s} = \frac{p(AV)_s}{RT_s}$$

$$= \frac{(14.7\ \text{lbf/in}^2)(200\ \text{ft}^3/\text{min})}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(580°\text{R})}\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right| = 13.69\ \text{lb/min}$$

and

$$m = \frac{pV}{RT} = \frac{(14.7)(20{,}00)\,|144|}{\left(\frac{1545}{28.97}\right)(515)} = 154\ \text{lb}$$

where the average of the initial and final temperatures is used to estimate the mass. Also, from A-20E; $k = 1.4$.

Continued on next slide

Problem 4-122 continued

Integrating Eq. (1) from $t = 0$ ($T = T_i = 40°F$) to any time t

$$-\left(\frac{m}{k \cdot \dot{m}_s}\right) \ln\left(\frac{T_s - T}{T_s - T_i}\right) = t$$

or, solving for T

$$T = T_s - (T_s - T_i)\exp\left[\left(-\frac{\dot{m}_s k}{m}\right)t\right]$$

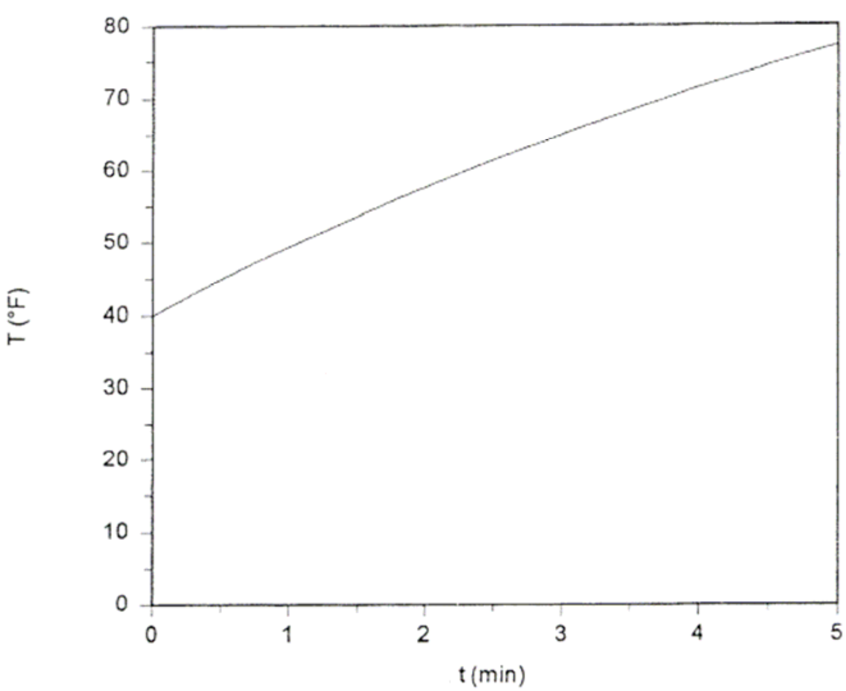
Inserting values

$$T = 120 - 80\exp\left[(-0.1245)t\right] \qquad (2)$$

Solving for t_2 when $T_2 = 70°F$ gives

$$t_2 = 3.79 \text{ min} \longleftarrow \qquad t_2$$

Eq. (2) can readily be plotted using software. The following plot was constructed using IT:

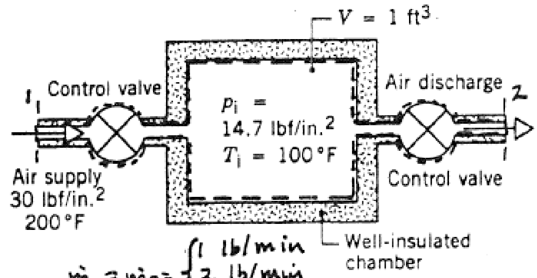


PROBLEM 4.123

<u>KNOWN</u>: Air flows through a well-insulated chamber. The initial conditions in the chamber, the supply conditions, and the inlet and exit mass flow rates are specified.

<u>FIND</u>: Determine the temperature and pressure of the air in the chamber as functions of time and plot for various mass flow rates.

<u>SCHEMATIC & GIVEN DATA</u>:



<u>ENGR. MODEL</u>: (1) For the control volume shown, $\dot{Q}_{cv} = 0$ and $\dot{W}_{cv} = 0$. (2) The temperature and pressure in the chamber are uniform throughout at any time. (3) Kinetic and potential energy effects are negligible. (4) The air is modeled as an ideal gas with constant specific heats.

<u>ANALYSIS</u>: The mass rate balance takes the form $dm_{cv}/dt = \dot{m}_1 - \dot{m}_2$. With $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$, $dm_{cv}/dt = 0$. Thus, the amount of mass contained within the control volume is constant. Using assumptions listed, the energy balance is

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}(h_1 - h_2)$$

or, with $U_{cv} = m_{cv}u$

$$m_{cv}\frac{du}{dt} + u\cancel{\frac{dm_{cv}}{dt}}^{\,0} = \dot{m}[h_1 - h(t)]$$

where $h_2 = h(t)$ by assumption (2). Further, $du = c_v dT$ and $h(t) = c_p T(t)$, so

$$m_{cv} c_v \frac{dT}{dt} = \dot{m}[h_1 - c_p T(t)]$$

or

$$\frac{dT}{dt} + \left(\frac{\dot{m}c_p}{m_{cv}c_v}\right)T = \frac{\dot{m}c_p T_1}{m_{cv}c_v}$$

Introducing $k = c_p/c_v$

$$\frac{dT}{dt} + \left(\frac{\dot{m}k}{m_{cv}}\right)T = \left(\frac{\dot{m}kT_1}{m_{cv}}\right)$$

The solution of this differential equation takes the form

$$T(t) = C\exp\left[-\left(\frac{\dot{m}k}{m_{cv}}\right)t\right] + T_1$$

With $T(0) = T_i$; $C = T_i - T_1$, so

$$T(t) = (T_i - T_1)\exp\left[-\left(\frac{\dot{m}k}{m_{cv}}\right)t\right] + T_1 \qquad (1)$$

Continued on next slide

Problem 4-123 continued

From the given data

$$m_{cv} = \frac{p_i V_i}{R T_i} = \frac{(14.7\ \text{lbf/in.}^2)(1\ \text{ft}^3)}{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(560^\circ\text{R})} \left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| = 0.0709\ \text{lb}$$

From Table A-20E ; $k \approx 1.4$. Also, $T_i = 100^\circ F$ and $T_1 = 200^\circ F$. Thus, from (1)

$\underline{\dot{m} = 1\ \text{lb/s}}$ $\quad T(t) = -100\, e^{-0.3291 t} + 200$

$\underline{\dot{m} = 2\ \text{lb/s}}$ $\quad T(t) = -100\, e^{-0.6582 t} + 200$

$\underline{\dot{m} = 5\ \text{lb/s}}$ $\quad T(t) = -100\, e^{-1.6455 t} + 200$

The pressure is $p = \frac{m R\, T(t)}{V}$. Thus

$$p = \left[\frac{(0.0709\ \text{lb})\ \frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}}{(1\ \text{ft}^3)\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|}\right][T(t) + 460]$$

Finally

$\underline{\dot{m} = 1\ \text{lb/s}}$ $\quad p(t) = -2.6258\, e^{-0.3291 t} + 17.3304$

$\underline{\dot{m} = 2\ \text{lb/s}}$ $\quad p(t) = -2.6258\, e^{-0.6582 t} + 17.3304$

$\underline{\dot{m} = 5\ \text{lb/s}}$ $\quad p(t) = -2.6258\, e^{-1.6455 t} + 17.3304$

Using IT to plot, we get